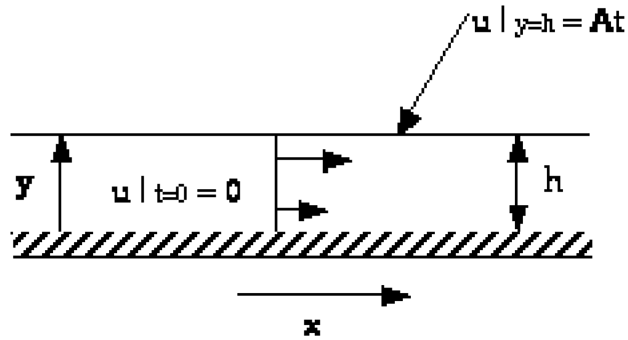
CHEG 60544 Transport Phenomena I
First Hour Exam

Closed Books and Notes

1). (20 points) Consider the startup flow depicted below. Fluid is confined in a gap of width h between a stationary lower wall and a moving upper wall. Initially the fluid is at rest, but at t = 0 the upper wall moves with a velocity proportional to time. You may take the flow to be unidirectional.

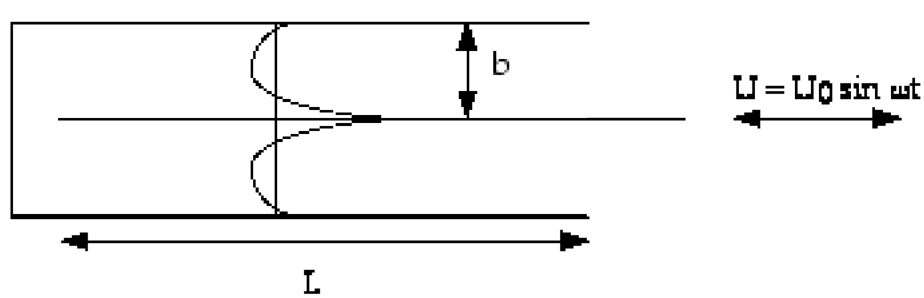


a). Write down the governing equation and boundary conditions and render them dimensionless. What is the characteristic time scale, and what is its physical interpretation?

b). Solve for the velocity distribution at large times.

c). Estimate how long we have to wait for this asymptotic velocity distribution to become valid.

2). (30 points) A simple viscous damper (shock absorber) is depicted below. The central plate moves with the oscillatory velocity $u = U_0 \sin \omega t$ and the outer shell is fixed. Because the end is closed, there can be no net flow (e.g., the integral of the velocity profile is zero at all times) and the motion of the plate will produce some pressure gradient $-\partial p/\partial x = G(t)$. We wish to determine the shear stress on the damper as a function of time. Since the length of the damper is much larger than the gap ($L/b >> 1$) we will assume time-dependent unidirectional flow. We also only need to consider the motion on one side, as that on the other will just be the mirror image.



a). Set up the problem, writing down all equations and boundary conditions. Render the problem dimensionless and show that it depends on a single dimensionless parameter.

b). Explicitly solve for the velocity profile, pressure gradient, and wall shear stress (on the damper) in the limit of low frequencies.

c). Perturbation expansion: Expand the velocity (and pressure!) as a regular perturbation expansion for small frequencies. Obtain the governing equation and boundary conditions to $O(\varepsilon)$, but **don't solve it** - it's fairly messy, albeit just a polynomial.

d. Recognizing that the **work** done on the damper plate is force times velocity, at what order in ε does inertia contribute to the damping effect (to leading order)?

e. Complete solution at all frequencies: Show how you would solve for the velocity distribution and pressure gradient for all frequencies (e.g., values of the dimensionless parameter of the problem which are O(1)). Solve the differential equation, but **don't** explicitly evaluate the coefficients or pressure gradient (again, they're messy!), but do explicitly show the equations they must satisfy.

3). (10 points) Index notation/linearity: A sphere is freely suspended (no force or torque) at the origin in an infinite fluid undergoing the general linear shear flow $u_i^\infty = A_{ij} x_j$ at zero Reynolds number.

a. Using index notation and the concept of linearity, prove that its translational velocity is zero.

b. Determine the most general relationship for the angular velocity Ω_i. (Hint: for a general linear shear flow A_{ij} this will be non-zero!)

c. The constant in part b is evaluated by picking "convenient" problems. The isotropic contribution may be evaluated by recognizing that in the flow $u_i^\infty = \varepsilon_{ijk} \Omega_j x_k$ (pure rotation) a freely suspended particle (of any shape) simply rotates with the fluid. Use this observation to get the unknown constant explicitly.

d. Using this result, determine the angular velocity of a sphere in the simple shear flow $u_i^\infty = \gamma \delta_{i1} x_2$, where γ is the shear rate.

e. This one (the body of revolution Jeffery's Orbit problem) was just too nasty - I'll give it to you guys for homework instead...