**CHEG 544 Transport Phenomena I**
**Second Hour Exam**

**Closed Books and Notes**

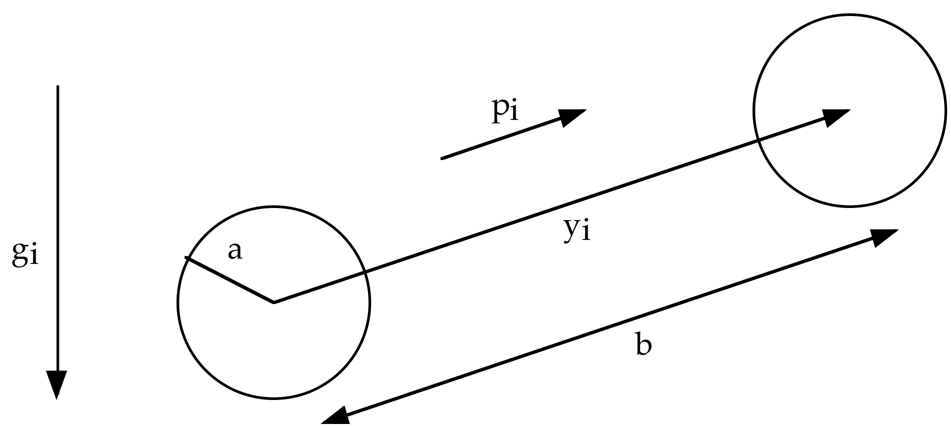
Problem 1). (10 points) Creeping Flow. A pair of identical spheres of radius a are settling under the influence of gravity in an infinite fluid as depicted below. At time t=0 they are separated by a vector $y_i = b\ p_i$ which is not in general aligned with the gravity vector $g_i$. In this problem we explore their dynamics under creeping flow conditions.

a). Using the concepts of linearity and reversibility, sketch out how the separation vector $y_i$ evolves in time (both magnitude b and direction $p_i$).

b). Again, from reversibility can there be a drift of the center of mass of the pair (e.g., the average of the two particle positions) in a direction which is not parallel to gravity?

c). How does the modification of the Stokes sedimentation velocity of each particle scale with the separation distance for separations much greater than the particle radius (e.g., $a/b << 1$)?

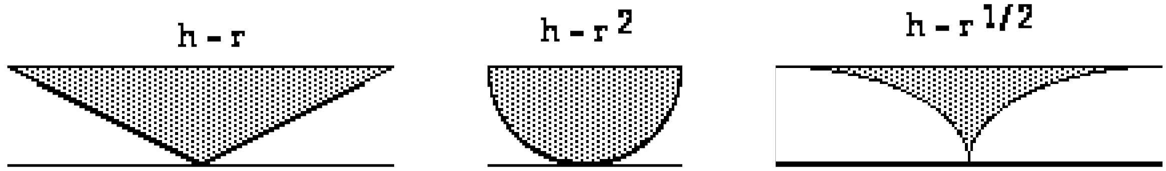
d). If the net gravitational force on each sphere is $F_k$, what is the velocity of the pair to first order in a/b? What is the ratio of their settling velocity to that at infinite separation if they are aligned side-by-side? If the separation vector $y_i$ is parallel to gravity?




Hint: Recall that the velocity field produced by a Stokeslet (point force singularity of strength $F_k$, the far-field of that produced by a sphere) is:

$$u_i = \frac{F_k}{8\pi\mu}\left\{\frac{\delta_{ik}}{r} + \frac{x_i\, x_k}{r^3}\right\}$$

Problem 2). (20 points) Lubrication: In class and for homework we have examined the detatchment of a variety of shapes from the surface of a plane. The nature of the lubrication singularity (e.g., the force when the separation distance at the center goes to zero) depends on the geometric dependence of the gap width on radius. For the case of a sphere, h varies as $r^2$, and the force is infinite (e.g., zero mobility at contact). For a cone h varies as r and the force is finite. In this problem we consider the general shape, where $h^* = (r/R)^{\alpha}$ in which $\alpha$ is a positive constant. For what values of $\alpha$ will the mobility at contact be non-zero? Note: the flat disk limit with zero separation at r = 0 corresponds to $\alpha = \infty$ rather than $\alpha = 0$.




You may find the r momentum equation useful to get the pressure gradient:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Note: If you solve the more complicated problem of finite separations at r = 0 (don't try this here!), it turns out that a particle can come into contact with a plane even if the mobility at contact is identically zero. This happens because for some values of $\alpha$ the lubrication singularity is integrable in time, even if infinite (zero mobility) at contact. **For an extra point**, can you tell me what the range of $\alpha$ is for which this curious behavior occurs? (Hint: the other bound is a problem we've already looked at...)

Problem 3). (20 points) Consider the two-dimensional problem depicted below. We are examining the flow pattern in produced by a pair of belts dragged toward a corner in a viscous fluid. The belts each move with a velocity -U, so the radial velocity boundary condition at $\theta = \pm\alpha$ is -U. This problem is closely related to the outer problem of the "sandwich" lubrication problem you solved for homework, and in this limit we take the net flow rate out the corner to be zero.

a). Determine the velocity profile using a streamfunction formulation.

b). How does the pressure distribution diverge as r approaches zero?

The velocities are given by:

$$u_\theta = -\frac{\partial\psi}{\partial r} \qquad ; \qquad u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geometry is given by:

$$\psi_\lambda = r^\lambda f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda-2)\theta) + D_\lambda \cos((\lambda-2)\theta)$$

We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\,\theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$

$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(\theta) + C_1\,\theta \sin(\theta) + D_1\,\theta \cos(\theta)$$

$$f_2(\theta) = A_2 + B_2\,\theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

Hint: Think about how the radial velocity and streamfunction have to depend on r, and determine the symmetry conditions in $\theta$!