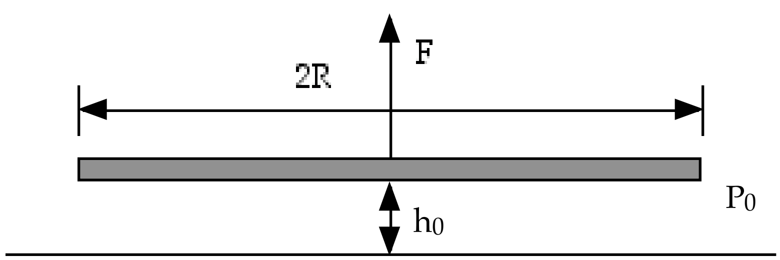
CBE 60544 Transport Phenomena I
Second Hour Exam

Closed Books and Notes

Problem 1). (10 points) Creeping Flow/Index Notation: Consider a sphere of radius a undergoing solid body rotation with angular velocity Ω_i in an unbounded fluid at rest. Using index notation, determine the disturbance velocity and pressure produced by the sphere at low Re. The following harmonics may be useful.

$$\frac{1}{r}\ ;\ \frac{x_i}{r^3}\ ;\ \left(\frac{x_i x_j}{r^5} - \frac{\delta_{ij}}{3r^3}\right)\ ;\ \left(\frac{x_i x_j x_k}{r^7} - \frac{x_i \delta_{jk} + x_j \delta_{ik} + x_k \delta_{ij}}{5r^5}\right)\ ;\ \ldots$$

Problem 2). (20 points) Lubrication: Consider the detatchment of a plate of radius R from a plane under the action of a force F, as depicted below.



The upward force on the disk given by F causes fluid to rush in to fill the gap, resulting in a reduced pressure that balances the force. If the pressure gets low enough (e.g., if the absolute pressure in the center reaches zero, or $p - p_0 = -p_0$) the fluid will cavitate and the disk will pop off pretty rapidly. Ignoring all buoyancy and gravitational effects, use lubrication theory to determine the maximum force F you can apply before cavitation occurs. You may find the r momentum equation useful to get the pressure gradient:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Problem 3). (20 points) Consider the two-dimensional problem depicted below. Two elastic belts separated by an angle 2α are in motion, such that each exerts a **constant shear stress** on the fluid in the radial direction drawing it towards the corner. Recall that the shear stress in cylindrical coordinates is given by:

$$\tau_{r\theta} = \mu \left[r \frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta} \right]$$

We wish to determine the velocity profile using a streamfunction formulation. The velocities are given by:

$$u_\theta = -\frac{\partial \psi}{\partial r} \quad ; \qquad u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geometry is given by:

$$\psi_\lambda = r^\lambda f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda\text{-}2)\theta) + D_\lambda \cos((\lambda\text{-}2)\theta)$$

We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\,\theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$

$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(\theta) + C_1\,\theta \sin(\theta) + D_1\,\theta \cos(\theta)$$

$$f_2(\theta) = A_2 + B_2\,\theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

Using the above relations, solve for the streamfunction and velocity distribution.

Hint: Think about how the radial velocity and streamfunction have to depend on r, and about symmetry relations for u_r and Ψ in θ.