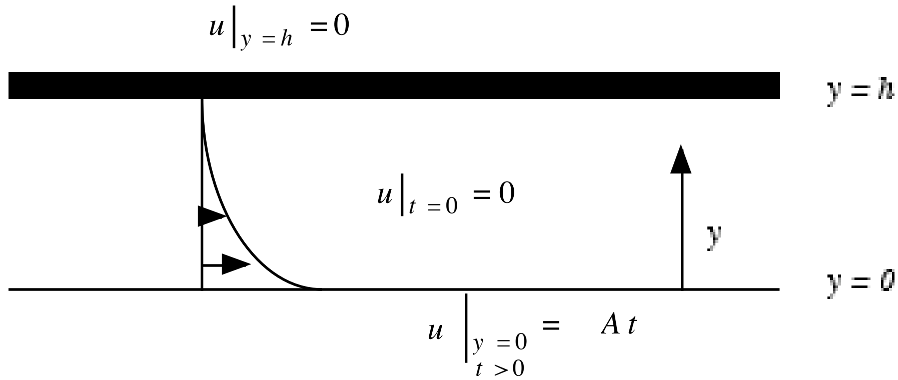
**CBE 60544 Transport Phenomena I**
**Final Exam**

**Closed Books and Notes**

Problem 1). (20 points) Unidirectional Flows/Scaling Analysis: The Couette viscometer depicted below is designed such that the velocity of the lower wall is linearly increasing in time.




a. Write down the differential equation and boundary conditions governing this problem, and render them dimensionless.

b. Determine the asymptotic velocity distribution at large times (e.g., everything except the part that exponentially decays in time). Explicitly determine the shear stress at the lower plate.

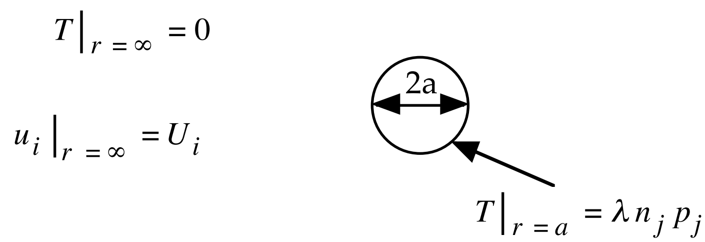
c. Solve for the decaying solution. You may leave the coefficients in terms of integrals.

d. At short times this problem admits a boundary-layer solution. Determine the similarity rule and variable, and transformed ODE and boundary conditions, but don't solve the ODE. What is the shear stress at the lower plate (to within an unknown constant)?

Problem 2). (20 points) Microfluidic Chip Design: You are assigned the task of determining the design parameters for a microfluidic biodetection system. Solute enters the square detection area of width and length L in a microchannel of depth d at a concentration $c_0$. The solute irreversibly binds to the surface detector at y=0, reducing the solute concentration at that wall to zero. Your design criteria are 1) process the solute at the fastest possible rate (e.g., highest possible total flow rate), 2) capture at least half of the solute in the incoming stream, and 3) keep the pressure drop in the detector less than some maximum $\Delta p_{max}$. For a fixed $\Delta p_{max}$, detector length (and width) L, solute diffusivity D and fluid viscosity $\mu$, estimate the optimum detector channel depth $d_{opt}$ and corresponding maximum solute flux which satisfies these criteria. Make any simplifying assumptions necessary to get a tractable problem.

Hint: the criteria (2) and (3) both place limitations on the maximum average fluid velocity $U_{max}$. The optimum depth will occur when these two limitations are equal.

Problem 3). (10 points) Matched Asymptotic Expansions. Consider a spherical thermal dipole ($T\big|_{r=a} = \lambda n_j p_j$ where $n_j$ is the local unit normal $x_j/r$ and $p_j$ is the director describing the dipole orientation) immersed in a uniform flow of magnitude U as depicted below. We wish to determine the influence of convection on the temperature distribution in the limit Re << Pe << 1 (e.g., small Pe, but zero Re so that the velocity distribution is known). Your mission is to determine the solution technique!




a. Starting with the convective diffusion equation given below, render the equation dimensionless and using regular perturbation expansions, set up a sequence of differential equations and boundary conditions at each order. Do this to as high an order as you think is appropriate.

$$u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^2}$$

b. Determine at what order we must resort to a singular perturbation expansion? What is the new length scale in the outer region?

Hint: the pure conduction solution is just the harmonic $T^{(0)} = \lambda a^2 \frac{x_i p_i}{r^3}$. Don't make this problem too difficult!

Problem 4). (10 points) Short Answer. Please answer the following questions briefly!

a. What is the Reynolds stress and how is it defined?

b. The axial spread of a solute slug in a microfluidic device is dominated by Taylor Dispersion. How does the dispersivity K vary with velocity U, molecular diffusivity D, and tube radius a? Don't worry about the constant - although it's nice if you can remember it...

c. Prove that turbulence must increase drag in flow through a pipe.

d. A body of revolution with orientation specified by a director $p_i$ is settling in a fluid at zero Re under the action of a force $F_i$. What is the most general relationship for its velocity $U_i$ as a function of orientation?

e. Does the answer to part d change at finite Re? Briefly justify your answer.