CHEG 60544 Transport Phenomena I
First Hour Exam

Closed Books and Notes

1). (15 points) A sphere is freely suspended (e.g., no forces or torques are exerted on it) in an infinite fluid undergoing the linear shear flow $u_i = A_{ij}\, x_j$ at zero Reynolds number.

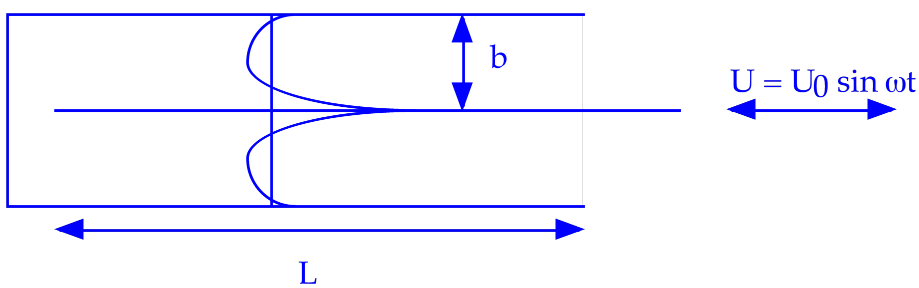
a. Determine the most general relationship for the sphere's angular velocity Ω_i as a function of the rate of strain tensor A_{ij}.

b. Using this, prove that the angular velocity of a sphere suspended in any pure straining flow (e.g., a symmetric rate of strain tensor $A_{ij} = A_{ji}$) is zero.

c. Evaluate the constant for arbitrary A_{ij} by recognizing that in solid body rotation the sphere will simply rotate with the fluid.

d. What is the angular velocity of a sphere in the simple shear flow $u_i = G\, \delta_{i1}\, x_2$? Be specific!

2). (20 points) A simple viscous damper (shock absorber) is depicted below. The central plate moves with the oscillatory velocity $u = U_0 \sin \omega t$ and the outer shell is fixed. Because the end is closed, there can be no net flow (e.g., the integral of the velocity profile is zero at all times) and the motion of the plate will produce some pressure gradient $-\partial p/\partial x = G(t)$. We wish to determine the shear stress on the damper as a function of time. Since the length of the damper is much larger than the gap ($L/b >> 1$) we will assume time-dependent unidirectional flow. We also only need to consider the motion on one side, as that on the other will just be the mirror image.



a). Set up the problem, writing down all equations and boundary conditions. Render the problem dimensionless and show that it depends on a single dimensionless parameter.

b). Explicitly solve for the velocity profile, pressure gradient, and wall shear stress (on the damper) in the limit of low frequencies.

c). Complete solution at all frequencies: Show how you would solve for the velocity distribution and pressure gradient for all frequencies (e.g., values of the dimensionless parameter of the problem which are O(1)). Solve the differential equation, but **don't** explicitly evaluate the coefficients or pressure gradient (they're messy!), but do explicitly show the equations they must satisfy.

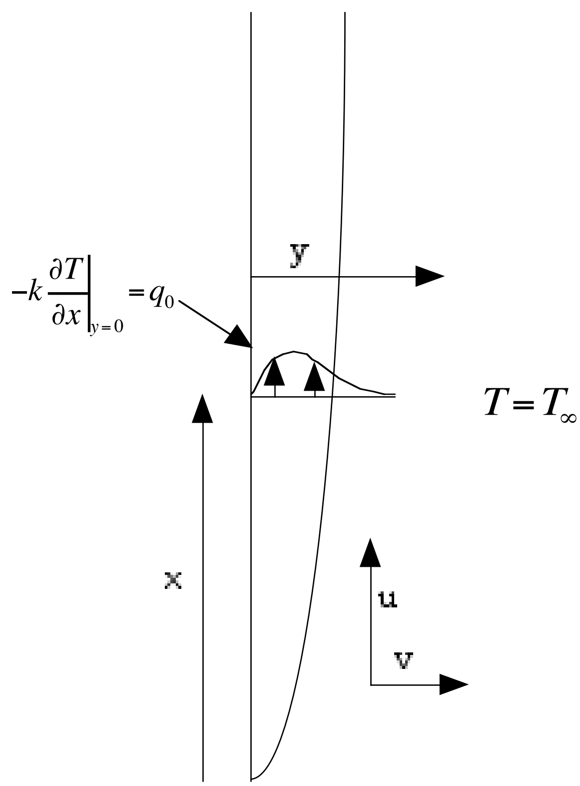
3). (20 points) Scaling analysis and self-similarity. For homework you scaled the equations governing natural convection from a point source of energy. In this problem we determine the characteristic velocity produced by the draft off of a heated plate as depicted below. We plate is giving off heat uniformly at a rate q_0, thus the boundary condition at y = 0 is:

The temperature at infinity is just T_∞. The velocities vanish at the plate, and u = 0 far from the plate.

a. Choosing L as the characteristic length in the x-direction, determine the characteristic thickness of the convection plume, the magnitude of the velocity, and the characteristic temperature of the plate. Remember: don't solve the equations, just scale them!

b. Using the results from part (a), or doing simple affine stretching again, show that the problem admits a self-similar solution and give the transformed variables in canonical form. You don't need to get the transformed differential equations, but determine the temperature of the plate as a function of x to within some unknown O(1) constant.

The flow is governed by:



$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \beta g\left(T - T_\infty\right)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u|_{y=0} = v|_{y=0} = u|_{y=\infty} = 0;$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{y=0} = q_0 \ ; \ T|_{y=\infty} = T_\infty$$