CBE 60544 Transport Phenomena I
Second Hour Exam

Closed Books and Notes

Problem 1). (15 points) Creeping Flow/Index Notation: Consider a sphere of radius a undergoing solid body rotation with angular velocity Ω_i in an unbounded fluid at rest.

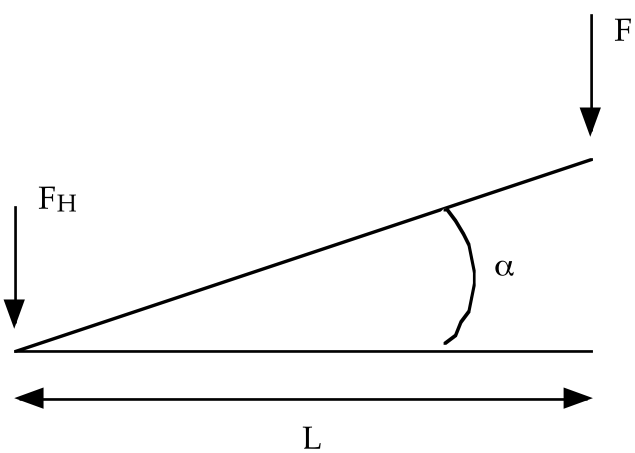
a. Using index notation and linearity, determine the torque exerted on the sphere to within an unknown constant.

b. Determine the constant. The following harmonics may be useful:

$$\frac{1}{r} \; ; \; \frac{x_i}{r^3} \; ; \; \left(\frac{x_i x_j}{r^5} - \frac{\delta_{ij}}{3r^3} \right) \; ; \; \left(\frac{x_i x_j x_k}{r^7} - \frac{x_i \delta_{jk} + x_j \delta_{ik} + x_k \delta_{ij}}{5r^5} \right) \; ; \; \ldots$$

Hint: Remember that the torque on an object is given by the surface integral of ε_{ijk} x_j σ_{kl} n_l dA (e.g., locally r X dF) where σ_{kl} is the stress tensor. Also, remember that at the surface of a sphere (where the integral is evaluated), x_i = a n_i where n_i is the unit normal and a is the radius. Note that the final integral for the torque will get a little messy: while I'd like you to get the "number", what I am primarily after is the procedure. Remember that because of symmetry and asymmetry, the vast majority of the terms cancel out or are easily evaluated!

Problem 2). (20 points) Lubrication: Consider the hinged plate depicted below.



The plate is forced downwards at the edge (x = L) with a force (per unit extension into the paper – it's a 2D problem) of magnitude F. The fluid squeezed out of the hinge resists this force, and thus it tries to "pop" the hinge. What force (per unit extension into the paper) F_H must the hinge withstand to prevent the hinge from popping? Solve the problem in the lubrication limit for small α. Hint: While the closing rate is determined by a torque balance about the hinge (remember the HW problem?), the total force on the hinge (F, F_H, and that exerted by the fluid being squeezed out) must balance as well…

Also, remember the integrals: $\int \ln x \, dx = x \ln x - x$; $\int x \ln x \, dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2$

Problem 3). (20 points) Streamfunctions: A viscous fluid is draining out of a narrow slit in a plane at a flow rate (per unit extension into the paper) of Q. Calculate the resulting velocity and pressure distributions.

Recall that the velocities are given by:

$$u_\theta = -\frac{\partial \psi}{\partial r} \qquad ; \qquad u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geometry is given by:

$$\psi_\lambda = r^{\lambda} f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda-2)\theta) + D_\lambda \cos((\lambda-2)\theta)$$

We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\,\theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$

$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(\theta) + C_1\,\theta \sin(\theta) + D_1\,\theta \cos(\theta)$$

$$f_2(\theta) = A_2 + B_2\,\theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

You may also find the radial momentum equation useful:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

$$+\,\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!