**CBE 60544 Transport Phenomena I**
**Final Exam**

**Closed Books and Notes**

Problem 1). (20 points) Spin coating – thick layers. A common way to coat a surface is to use *spin coating*, a technique in which a layer of fluid is deposited on a surface (here a disk of radius R) which is then spun at angular velocity Ω. Centrifugal force causes the fluid to be thrown out radially, leading to the layer getting thinner over time. Because of the way the problem works, however, the flow is always pseudo-steady-state: you don't have to worry about the time derivative in the equations of motion! The problem is also axisymmetric, eliminating all the θ derivatives as well.

For this problem we will look at the limit of initial thinning where the fluid layer is much thicker than the boundary layer next to the rotating plate where all the motion occurs. Scale the equations and use them to determine both the boundary layer thickness and the characteristic velocity in the z direction. This will determine the rate at which the layer thins out. Note that you would have to get this thinning velocity numerically, but if you have scaled everything correctly you get it to within some O(1) factor without solving anything! This solution would be valid until the layer thickness h approaches the boundary layer thickness δ.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho\, r\, u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho\, u_\theta) + \frac{\partial}{\partial z}(\rho\, u_z) = 0$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, u_r)\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_r\, u_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, u_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z$$

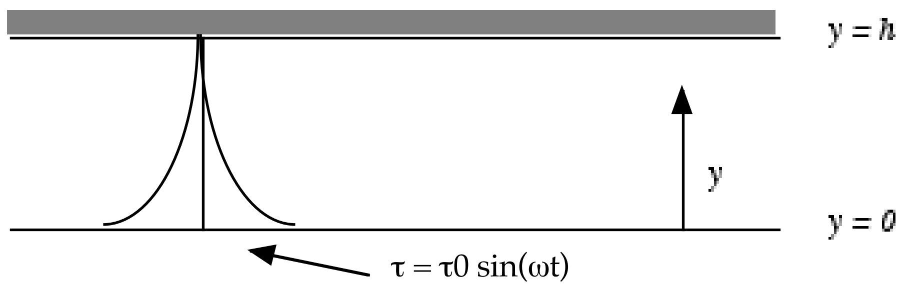
Problem 2). (20 points) Spin coating – thin layers. After some time, the fluid layer of problem 1 thins out until it is thinner than the boundary layer thickness. At this point the velocity profile in the θ direction is just uniform rotation $u_\theta = \Omega r$ for all z. Centrifugal force still throws the fluid out radially and causes the fluid to continue to thin out, however, and thus the film depth h continues to decrease. In this ***lubrication limit***, explicitly determine how the film thickness h varies with time starting from some intial thickness $h_0$.

Problem 3). (20 points) Oscillatory flows. Consider the plane Couette flow depicted below. The lower plate is subjected to an oscillatory shear stress $\tau = \tau_0 \sin(\omega t)$, and the upper plate is fixed (u = 0). Solve for the following:

a. Write down the governing equation and boundary conditions and render them dimensionless (use the low frequency scalings). Show that the problem depends on a single dimensionless parameter.

b. Explicitly determine the amplitude of motion of the lower plate in the limit of low frequencies.

c. Rescale the problem for high frequencies, and again explicitly determine the amplitude of the motion of the plate in this limit.




Problem 4). (10 points) Short Answer. Please answer the following questions briefly!

a. If a three-dimensional object is freely suspended (no net force or torque) in creeping flow, how does the disturbance *pressure* decay with r?

b. If the same object has a net force applied to it, how does the disturbance *velocity* decay with r?

c. How is the Reynolds stress defined and what does it represent?

d. Under what conditions can a flow be irrotational?

e. We can define a turbulent Prandtl number $\nu^{(t)}/\alpha^{(t)}$ (e.g., the ratio of momentum and thermal diffusivities due to turbulence). What can you say *a priori* about this quantity?