CBE 60544 Transport Phenomena I
First Hour Exam

Closed Books and Notes

1). (10 points) Index Notation/Creeping Flow: Prove that the angular velocity of a sphere freely suspended in pure straining flow (e.g., a linear shear flow with symmetric rate of strain tensor) is identically zero under creeping flow conditions.

2). (25 points) Unidirectional Flow: An oscillatory syringe pump is used to drive sinusoidal oscillatory flow in a capillary of radius a. The tidal volume of the syringe pump is V_0 (e.g., the total volume of the plunger displacement). Your goal is to determine the pressure gradient in the capillary, given the fluid has some density ρ and viscosity μ.

a. Scale the problem (differential equation and all boundary conditions – including the integral one!) assuming unidirectional flow and show that the dimensionless amplitude of the pressure gradient is a function of a single dimensionless group. Use the viscous limit scaling.

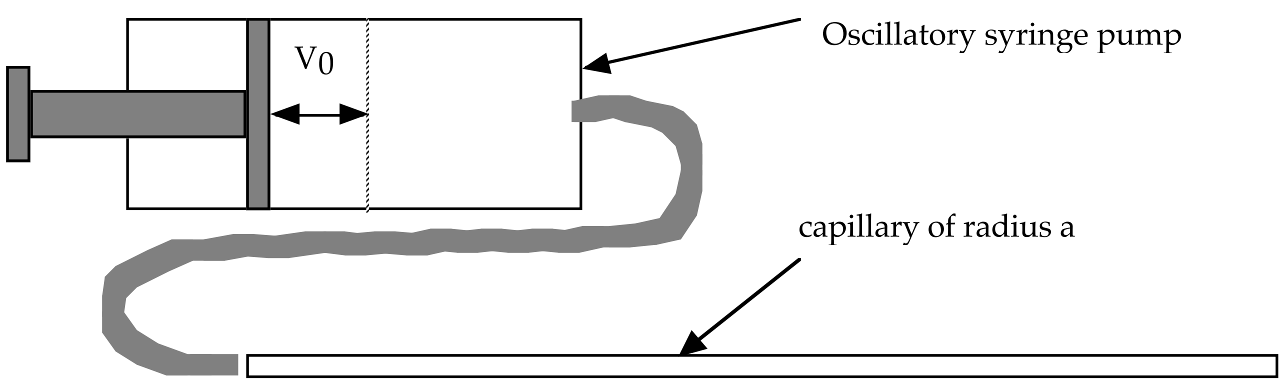
b. Solve for the amplitude of the pressure gradient at low frequencies.

c. Show how you solve for the amplitude of the pressure gradient at all frequencies (e.g., where the dimensionless parameter in part (a) is of O(1)), reducing the problem to the appropriate ODE and boundary conditions. You do not need to solve the ODE (unless you really love Bessel functions), but show me that you know how to solve the problem!

The following equation should be useful:

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z$$

Remember that the vast majority of these terms are zero for unidirectional flow!



3). (25 points) Mass Transfer in Falling Films: A common application of falling liquid films is in gas-liquid stripping or absorption. Consider the system depicted below. A fluid film of thickness δ, density ρ, and viscosity μ is falling down a stationary wall under the influence of gravity. The liquid, which initially has no dissolved solute ($c|_{z=0} = 0$), is exposed to a gas, which raises the concentration at the surface of the film to c_{eq} (e.g., $c|_{y=\delta} = c_{eq}$). The solute diffuses into the liquid as the film flows downwards, and is assumed not to influence the velocity profile. There is no mass transfer through the wall at y=0.

a. Determine the fluid velocity distribution and average velocity U. Assume unidirectional flow and a stress-free gas-liquid interface.

b. Render the mass transfer problem dimensionless. Using the separation of variables approach, show how the concentration distribution can be calculated. Note that the (correct) resulting Sturm-Liouville problem has no analytic solution, so don't try to get it! Instead, completely set everything up, including explicitly showing how the coefficients of the series solution should be calculated.

c. Quantitatively estimate the distance the film must travel downwards before equilibration is approached by approximating the actual velocity profile with a uniform (y-independent) velocity profile equal to the average velocity from part (a) (hint: get the lead eigenvalue of this simpler problem!). How does this distance depend on the viscosity of the fluid and the thickness of the film?

The equation governing the mass transfer is:

$$u\frac{\partial c}{\partial z} = D\frac{\partial^2 c}{\partial y^2} \quad ; \quad \left.\frac{\partial c}{\partial y}\right|_{y=0} = 0 \quad ; \quad c|_{y=\delta} = c_{eq} \quad ; \quad c|_{z=0} = 0$$

Note that in general u is NOT U, and is a function of y (determined in part a)! We only approximate it with a uniform profile in the last bit for estimation purposes.