CBE 60544 Transport Phenomena I
Final Exam

Closed Books and Notes

Problem 1). (20 points) Spin coating – thick layers. A common way to coat a surface is to use *spin coating*, a technique in which a layer of fluid is deposited on a surface (here a disk of radius R) which is then spun at angular velocity Ω. Centrifugal force causes the fluid to be thrown out radially, leading to the layer getting thinner over time. Because of the way the problem works, however, the flow is always pseudo-steady-state: you don't have to worry about the time derivative in the equations of motion! The problem is also axisymmetric, eliminating all the θ derivatives as well.

For this problem we will look at the limit of initial thinning where the fluid layer is much thicker than the boundary layer next to the rotating plate where all the motion occurs. **Scale the equations and use them to determine both the boundary layer thickness and the characteristic velocity in the z direction.** This will determine the rate at which the layer thins out. Note that you would have to get this thinning velocity numerically, but if you have scaled everything correctly you get it to within some O(1) factor without solving anything! This solution would be valid until the layer thickness h approaches the boundary layer thickness δ (which you have calculated).

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho\, r\, u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho\, u_\theta) + \frac{\partial}{\partial z}(\rho\, u_z) = 0$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, u_r)\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, u_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z$$

Problem 2). (20 points) Spin coating – thin layers. After some time, the fluid layer of problem 1 thins out until it is thinner than the boundary layer thickness. At this point the velocity profile in the θ direction is just uniform rotation $u_\theta = \Omega r$ for all z. Centrifugal force still throws the fluid out radially and causes the fluid to continue to thin out, however, and thus the film depth h continues to decrease. In this ***lubrication limit***, explicitly determine how the film thickness h varies with time starting from some initial thickness h_0.

Problem 3). (20 points) A fluid is flowing through a 2-D channel of width 2b with average velocity U as depicted below. It is undergoing a chemical reaction which releases heat uniformly at a rate S (e.g., energy/volume), and is cooled at both walls to the initial temperature T_0. The velocity profile is well-developed (e.g., while a function of y, it is not a function of x), and the fluid has a heat capacity ρC_p and thermal conductivity k. Solve for the following:

a. Render the equations dimensionless and determine the characteristic scaling for the temperature. Show under what conditions we can neglect diffusion of energy in the flow direction.

b. Determine the asymptotic temperature distribution at large x.

c. Quantitatively estimate how far downstream (e.g., distance in x) we must travel for the solution in b to become valid.

d. Still neglecting diffusion in the flow direction, solve for the temperature distribution near the centerline for small x (e.g., not too close to the walls). You may find rescaling the problem in this region to be useful, but it is not necessary.

e. Using the asymptotic results of b, c, and d, give a dimensionless plot of the centerline temperature as a function of distance down the channel.

$$\rho C_p u \frac{\partial T}{\partial x} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + S$$