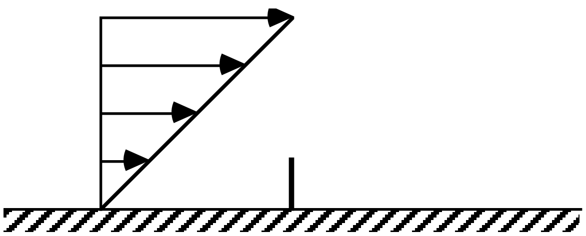
CHEG 544 Transport Phenomena I
Second Hour Exam

Closed Books and Notes

1). (10 points) Consider creeping shear flow past the baffle depicted below. Using your intuition sketch the expected streamlines, paying particular attention to any areas of recirculation (eddies) and symmetry.



2). (15 points) A fundamental singularity of Stokes flow (not **the** singularity of a point force) is the flow due to a point **source** of fluid.

a. If fluid is introduced at the origin at some volumetric flow rate Q, write down the velocity distribution in index notation. Hint: the flow will be purely radial ($\mathbf{e}_r = x_i/r$ in index notation) and it must satisfy continuity.

b. Using the equation of motion in index notation, determine the pressure distribution for source flow. Hint: the distribution is *extremely* simple. What is the normal stress component σ_{rr}?

3). (25 points) While general straining motion past a liquid drop in index notation is a bit too difficult for us to solve on an exam, let's look at the related problem of uni-axial extensional flow past a drop.

a. (most of the points) Consider uni-axial extensional flow (i.e., the undisturbed streamfunction is given by $\psi = -2Gr^3Q_2(\eta)$ where G is the shear rate and $Q_2(\eta) = 1/2\ \eta\ (\eta^2 - 1)$) around a liquid drop of radius a. If the surrounding fluid has a viscosity μ and the liquid in the drop has a viscosity $\lambda\mu$, write down the form of the streamfunction in the two regions together with **all** boundary conditions. Determine which constants are non-zero.

b. (the rest of the points) Obtain explicit values for the constants by using the boundary conditions. The contribution of the droplets to the stress in the fluid may be determined from the limiting form of the disturbance streamfunction far from the drop (don't do this). The relative viscosity of a dilute suspension of spherical drops is given by $\eta = \mu\ (1 + 1/2\ C_2\ \phi)$ where ϕ is the volume fraction of the drops and C_2 is in dimensionless form. Use this to determine the dependence of the relative viscosity on the drop viscosity ratio λ.

Recall:

$$\psi = \sum_{n=1}^{\infty} \left[A_n\, r^{n+3} + B_n\, r^{n+1} + C_n\, r^{2-n} + D_n\, r^{-n}\right] Q_n(\eta) \quad \text{where } \eta = \cos\theta$$

and:

$$u_r = \frac{-1}{r^2}\frac{\partial\psi}{\partial\eta}\ ,\quad u_\theta = \frac{-1}{r}\frac{\partial\psi}{\partial r}\frac{1}{\sqrt{1-\eta^2}}\ ,\quad \tau_{r\theta} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial\theta}\right]$$

hence:

$$\tau_{r\theta} = \frac{-\mu}{\sqrt{1 - \eta^2}} \left[r \frac{\partial}{\partial r}\left(\frac{\psi_r}{r^2}\right) - \frac{1}{r^3}\, \psi_{\eta\eta} \right]$$