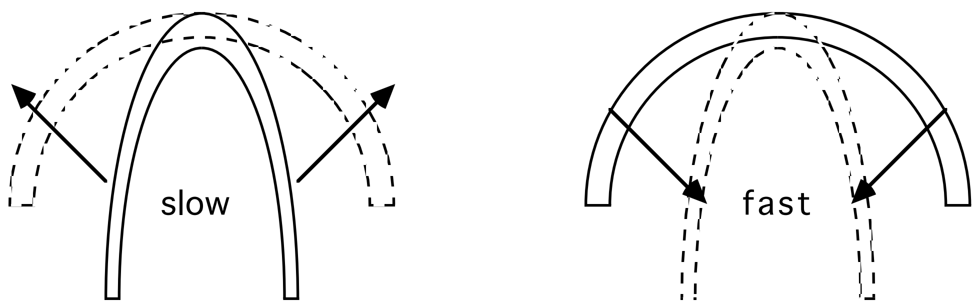
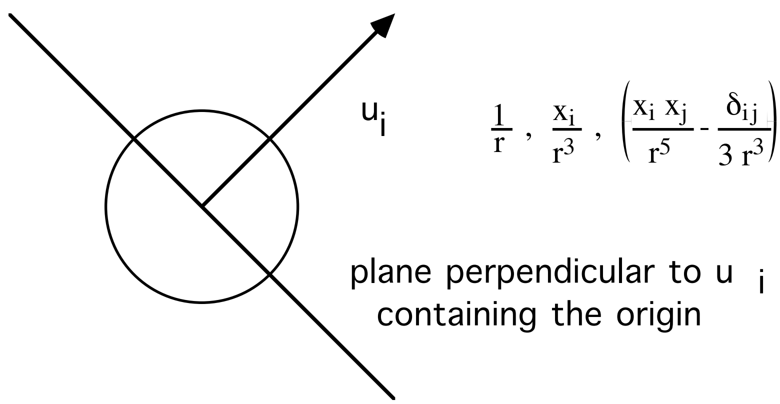
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes

1). (10 points) A jellyfish swims by drawing fluid into its umbrella-like body (see the diagram below) slowly and then ejecting it with high velocity, propelling it forwards. Would this still work if the jellyfish were of microscopic dimensions (consider the dimensionless velocity of the jellyfish here)? Briefly explain your answer.



2). (20 points) Using arguments based on index notation and the nature of vectors and tensors, determine how rapidly the disturbance pressure produced by a sphere translating with uniform velocity u_i falls off as r (the distance from the sphere) becomes large. What is the pressure distribution in the plane perpendicular to the direction of motion which contains the center of the sphere? The first few harmonics are given below for your convenience.



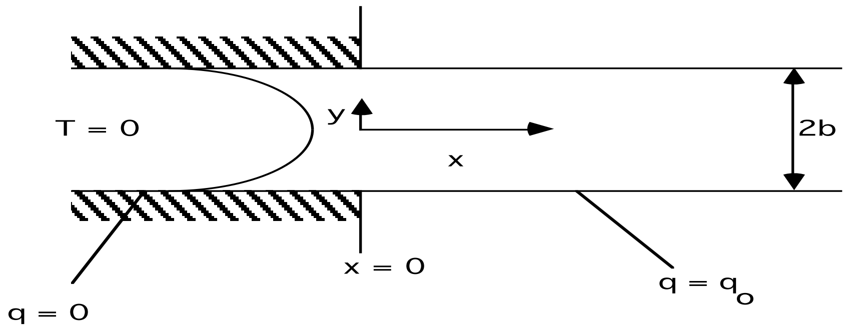
$$\frac{1}{r}\ ,\ \frac{x_i}{r^3}\ ,\ \left(\frac{x_i\, x_j}{r^5} - \frac{\delta_{ij}}{3\, r^3}\right)\ ,\ \ldots$$

3). (20 points) In electrophoresis an electric field is used to cause charged particles or molecules to experience some drift across fluid streamlines. By manipulating the charge or mobility of the molecules of interest, this process can be used as a separation technique. If the flux of a dilute species A with molar concentration c_A relative to that due to pure convection is given by:

$$J_A = \lambda\, c_A\, \tilde{\mathbf{E}} - D\, \tilde{\nabla}\, c_A$$

develop an integral conservation equation governing the time dependent concentration distribution. Applying the divergence theorem, convert this to a differential equation valid everywhere in the flow. Be sure to include the effects of a fluid velocity profile **u** and remember that the electric field **E** may be a function of position and time.

4. (50 points total) Many analogies exist between energy and momentum transfer. In this problem we will look at the temperature distribution acquired by a fluid in a channel as it passes from an insulated region through a region where there is a constant heat flux into the fluid at the walls. Throughout this problem we will assume that the velocity distribution is fully developed (parabolic). The differential equation governing the temperature distribution and the boundary conditions are given below.



a. (10 points) Render the problem and boundary conditions dimensionless. Show, by rendering x dimensionless such that x-convection balances y conduction, that for sufficiently high velocities conduction in the x-direction is negligible.

$$u_i \frac{\partial T}{\partial x_i} = \alpha \frac{\partial^2 T}{\partial x_i^2} \quad , \quad -k\, n_i \frac{\partial T}{\partial x_i}\bigg|_{\substack{x_2 = \pm b \\ x_1 < 0}} = 0 \quad , \quad -k\, n_i \frac{\partial T}{\partial x_i}\bigg|_{\substack{x_2 = \pm b \\ x_1 \geq 0}} = q_0 \qquad \begin{array}{l} x_1 \equiv x \\ x_2 \equiv y \end{array}$$

b. (20 points) Neglecting conduction in the x-direction solve for the temperature distribution. Recall that you must first solve for the asymptotic solution at large distances down the channel (because of the constant heat flux this will still be a function of x, where the final constant in the solution is determined from an energy balance on the channel rather than an initial condition at x=0) and then you may obtain an eigenfunction solution for the decaying solution. Obtain an explicit equation for the coefficients of this series solution, but do not evaluate the resulting integral.

c. (20 points) For small values of x (and values of y close to the wall) it is simpler to solve for the temperature distribution via a self-similar solution to the boundary layer equation. To solve for the temperature profile in this region, let us consider the flow at the upper wall and define a coordinate s=b-y. Show that if we neglect conduction in the x direction (as before) and approximate the velocity profile by the linear shear flow $u=\gamma s$ (its limiting form as $s \to 0$) then the problem will admit a similarity solution. Obtain the similarity rule, similarity variable, and transformed ODE with corresponding boundary conditions (in canonical form please) but do not solve the resulting problem. How does the temperature at the wall and the thickness of the thermal boundary layer vary with x?