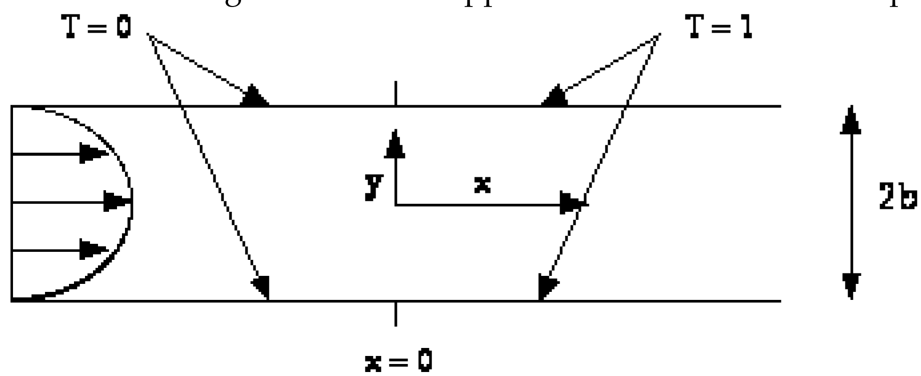
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes

First Problem Counts Double

Problem 1). Most of this semester we have examined the asymptotic solution to complex problems. In this problem you will examine convective heat transfer in pressure driven flow through a channel. Suppose we have the channel depicted below:



For all $x < 0$ the walls are maintained at a temperature $T = 0$, and for $x > 0$ at a temperature $T = 1$. The fluid velocity at the centerline is U, the width of the channel is 2b, and the thermal diffusivity is α. If we assume constant properties everywhere, the problem is governed by the differential equation:

$$u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^2}$$

where the velocity is just unidirectional Poiseuille flow.

I want you to solve the following problems:

a. The asymptotic temperature distribution for large x (this is *simple*).

b. The eigenvalue decaying solution.

c. The self-similar boundary layer solution near the entrance to the heated section.

d. The conduction dominated regime near the boundary condition discontinuity.

In each case I want you to carefully determine the domain of validity of the solution.

Hint: Laplace's equation in cylindrical coordinates (2-D) is:

$$\nabla^2 T \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0$$

Problem 2). In a recent homework you showed that boundary layer growth was retarded by an accelerating flow. In this problem we look at the opposite case: a decelerating flow. Consider flow past a flat plate where the x-velocity outside the boundary layer is given by $u_e = \lambda\, x^{-1/2}$.

a. Render the Navier-Stokes equations dimensionless for this problem and determine the conditions under which a boundary layer description is valid.

b. Using simple affine stretching, show that the boundary layer equations admit a self-similar solution, and determine the rate of boundary layer growth with x.

Problem 3). Using the techniques learned in class, determine the velocity and pressure distribution for a sphere of radius a rotating with a constant angular velocity $\Omega \mathbf{j}$ in a fluid at rest under creeping flow conditions. This is easy if you make maximum use of the principle that the double dot product of symmetric and antisymmetric tensors is zero.