CHEG 544 Transport Phenomena I
Second Hour Exam

Closed Books and Notes

1). (10 points) Consider the flow due to a point source of fluid, such that the fluid emanates from the origin at a rate Q (vol/time -- a scalar). What is the resulting pressure and velocity distribution? There is obviously more than one way to solve this problem, but for full credit you should do it using the index notation approach we have been working with in class.

2). (20 points) Consider creeping flow in a slot of infinite length as depicted below. The flow is driven by a lid moving with velocity U, and the walls of the slot ($x = \pm a$) may be considered to be stress free. There is no flow through the boundaries of the slot, so the walls are a streamline. Solve for the velocity profile in terms of the streamfunction.

Hint: the velocity and the streamfunction should die away as $y \to \infty$. This should suggest what functional representation you should take for the y dependence of the streamfuction (e.g., trigonometric, hyperbolic, or exponential).

3). (20 points) Boundary layer flow past a flat plate at zero incidence (depicted below) is governed by the boundary layer equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

with boundary conditions:

$$u\big|_{y \to \infty} = u\big|_{x=0} = U \;\;,\;\; u\big|_{y=0} = v\big|_{y=0} = 0$$

where ν is the kinematic viscosity, and u and v are the velocities along the plate (x direction) and normal to the plate (y direction), repectively. In terms of the streamfunction we have the equivalent expression:

$$\psi_y \psi_{xy} - \psi_x \psi_{yy} = \nu\, \psi_{yyy}$$

and:

$$\psi_y\big|_{y \to \infty} = \psi_y\big|_{x=0} = U \;\;,\;\; \psi_y\big|_{y=0} = \psi_x\big|_{y=0} = 0$$

Render these equations dimensionless and show that they admit a self-similar solution accessible through simple affine stretching. Obtain the similarity rule and variable in canonical form, as well as the new ODE and boundary conditions. This will be the famous Blasius equation for flow past a flat plate.