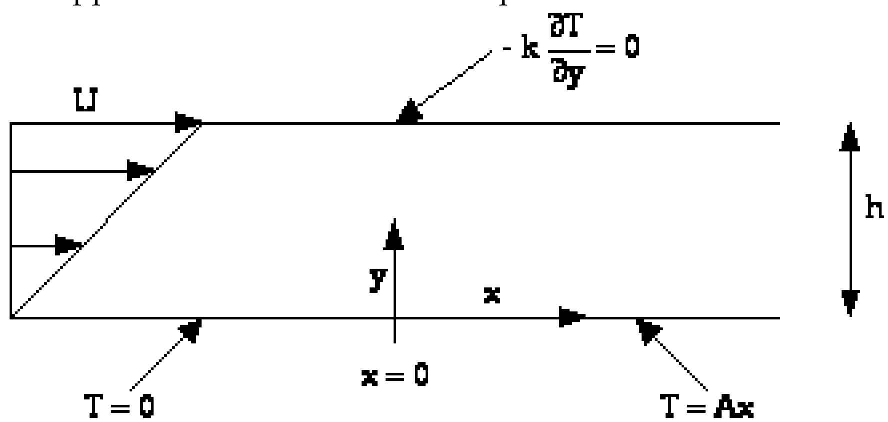
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes
First Problem Counts Double

Problem 1). Most of this semester we have examined the asymptotic solution to complex problems. In this problem you will examine convective heat transfer in plane Couette flow. Suppose we have the channel depicted below:



For all x < 0 the lower wall is maintained at a temperature T = 0, and for x > 0 at a linearly increasing temperature T = Ax where A is a constant. The upper wall is insulated, so that there is no heat flux through it. The velocity of the upper wall is U, the width of the channel is h, and the thermal diffusivity is α. If we assume constant properties everywhere, the problem is governed by the differential equation:

$$u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^2}$$

where the velocity is just unidirectional plane Couette flow.

I want you to solve the following problems, in each case carefully determining the domain of validity of the solutions:

a. The asymptotic temperature distribution for large x (this is simple, but not trivial).

b. The eigenvalue decaying solution. Set up the Sturm-Liouville eigenvalue problem completely, and show how to calculate the coefficients, but don't actually solve the DE for the eigenfunctions. It turns out that, from Gradshteyn and Ryzhik, the eigenfunctions can be found in terms of Bessel functions of order 1/3.

c. The self-similar boundary layer solution near the entrance to the heated section. Again, set up the problem completely in canonical form obtaining the ODE and boundary conditions. How does the heat flux at the wall depend on x?

Problem 2). Lubrication: Consider the squeeze flow between two disks of radius R separated by a distance h. The disks are approaching each other with a velocity dh/dt = -V (a constant). Calculate the lubrication force resisting this motion. Hint: this problem is virtually identical to the squeeze flow between a sphere and a plane, except here the geometry is *much* simpler - the gap h is not a function of r! You may find the r momentum equation useful to get the pressure gradient:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Problem 3). Consider a sphere suspended in the general linear shear flow $u_i = \Gamma_{ij}\, x_j$ under creeping flow conditions. If the sphere is free to rotate, what is its angular velocity Ω_i? Hint: Use linearity, and evaluate the coefficient for a particular flow field for which you know the answer (e.g., pure rotational flow).