Closed Books and Notes

1). (15 points) A sphere is a distance h from a plane and is acted on by a force F_i. We perform two experiments. In the first experiment the force F is applied perpendicular to the plane, and the sphere is observed to move with a velocity U_a. In the second the force is applied parallel to the plane, and the observed velocity is U_b. Using this, determine the velocity for an arbitrary applied force. Hint: the velocity will be a function of both F_i and n_i, the unit normal to the plane.

2). A commonly used viscosity measurement tool is the controlled stress rheometer, in which the applied stress is controlled and the resulting motion of a plate is used to calculate the viscosity. A simplified version of such a system is depicted below:
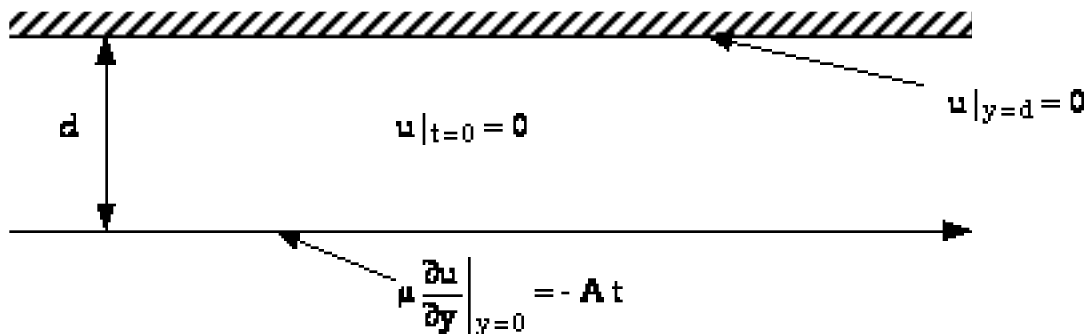
In this problem we are measuring the viscosity by looking at the effect of a linear ramp in the shear stress. The entire system is initially at rest, and at time t = 0 a shear stress given by τ = - A t is applied to the lower wall. The velocity U of the lower wall is measured as a function of time, with the apparent viscosity being defined as $\mu_{app} = \tau\, d\, /\, U$ where d is the gap width.

a). (15 points) If the fluid is actually Newtonian with constant viscosity, how will the apparent viscosity depend on time and the other parameters in the problem for large times? When will this solution be valid?

b). (15 points) The solution obtained in part (a) breaks down at short times. Solve for the velocity at these shorter times using separation of variables. Obtain the eigenfunctions and eigenvalues, and show how the constants in the series solution would be obtained.

c). (15 points) For very short times the problem may admit a similarity solution. Using simple affine stretching, show that the problem admits such a similarity solution, and give the similarity rule, similarity variable, transformed differential equation, and boundary conditions in canonical form. Determine how the apparent viscosity depends on the parameters in the problem to within some unknown constant. When will this solution be valid?