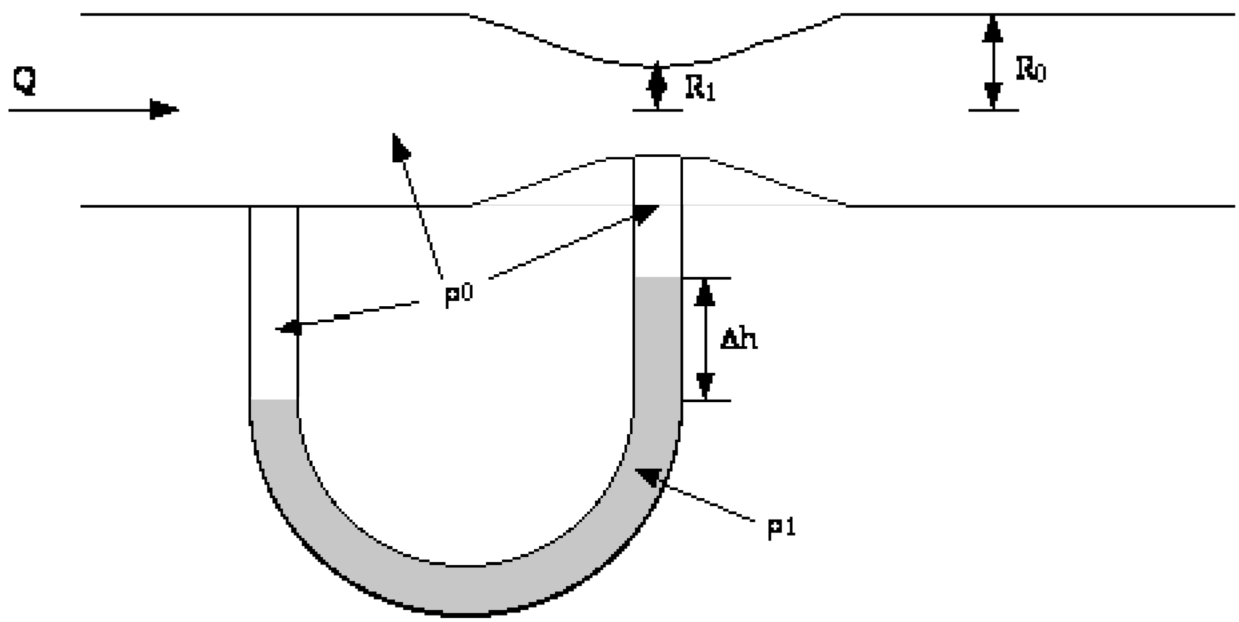
CHEG 544 Transport Phenomena I
Second Hour Exam

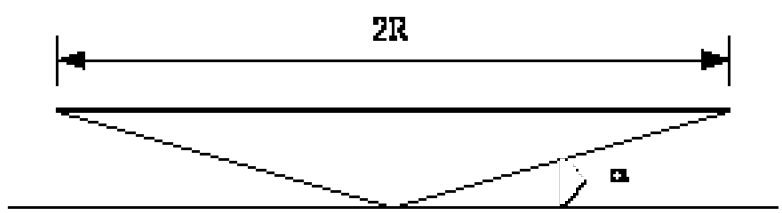
Closed Books and Notes

Problem 1). (10 points) For high Re flow (e.g., inviscid, irrotational potential flow) a con striction can be used to estimate the flow rate through a pipe. Consider the pipe flow d escribed below:



Using Bernoulli's equation, determine the flow rate Q (volume/time) for some measure d Δh in the U tube manometer.

Problem 2). (20 points) Lubrication: A cone of radius R and angle α is initially in contact with a plane as depicted below.



Using lubrication theory, determine the force F necessary to pull the cone off the plane with some velocity V in the limit of small α. You may find the r momentum equation u seful to get the pressure gradient:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms sur vive!

Problem 3). (20 points) Consider the two-dimensional problem depicted below. Fluid d rains from a trough with interior angle 2a at a rate Q/W where W is the extension of th e trough in the third dimension. We wish to determine the velocity profile using a strea mfunction formulation. The velocities are given by:

$$u_\theta = -\frac{\partial \psi}{\partial r} \quad ; \quad u_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geom etry is given by:

$$\psi_\lambda = r^{\lambda} f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda-2)\theta) + D_\lambda \cos((\lambda-2)\theta)$$

We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\,\theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$
$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(2\theta) + C_1\,\theta \sin(\theta) + D_1\,\theta \cos(\theta)$$
$$f_2(\theta) = A_2 + B_2\,\theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

Using the above relations, solve for the streamfunction and velocity distribution. Hint: Think about how the radial velocity and streamfunction have to depend on r.