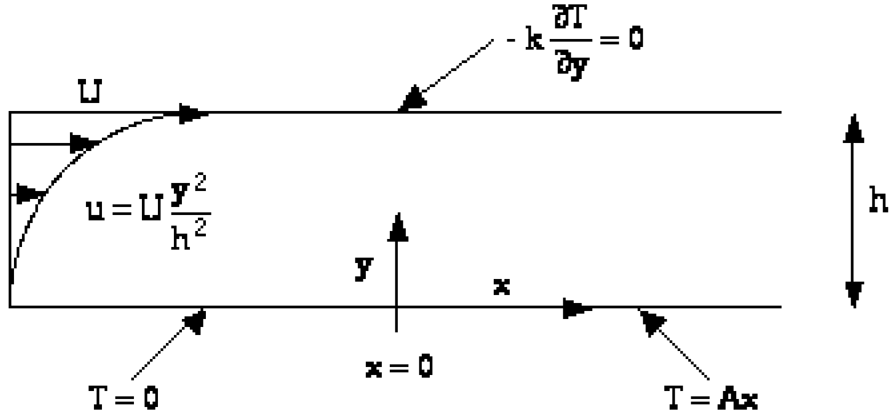
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes

Problem 1). Most of this semester we have examined the asymptotic solution to complex problems. In this problem you will examine convective heat transfer in a combined plane Couette and Poiseuille flow. Suppose we have the channel depicted below. The upper wall moves with a velocity U and the lower wall is fixed. In addition to the shear flow there is a pressure driven backflow resulting in the purely quadratic dependence of velocity on y (e.g., the shear rate at the lower wall is zero):



For all $x < 0$ the lower wall is maintained at a temperature $T = 0$, and for $x > 0$ at a linearly increasing temperature $T = Ax$ where A is a constant. The upper wall is insulated, so that there is no heat flux through it. The width of the channel is h, and the thermal diffusivity is α. If we assume constant properties everywhere, the problem is governed by the differential equation:

$$u_j\frac{\partial T}{\partial x_j}=\alpha\frac{\partial^2 T}{\partial x_j^2}$$

I want you to solve the following problems, in each case carefully determining the domain of validity of the solutions:

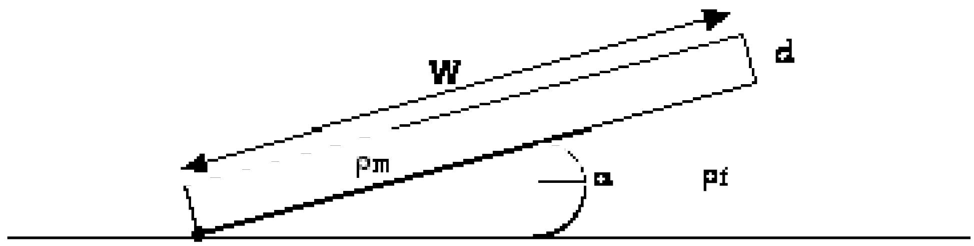
a. The pressure gradient required to produce the desired flow.

b. The asymptotic temperature distribution for large x.

c. The eigenvalue decaying solution. Set up the Sturm-Liouville eigenvalue problem completely, and show how to calculate the coefficients, but don't actually solve the DE for the eigenfunctions.

Problem 2). For the geometry and flow field depicted in problem 1, obtain the self-similar boundary layer solution near the entrance to the heated section. Again, set up the problem completely in canonical form obtaining the ODE and boundary conditions, but don't solve the resulting ODE. In particular determine how the heat flux at the wall and boundary layer thickness depend on x.

Problem 3). Lubrication: A plate of density ρ_m, thickness d and width W (you may consi der the third dimension to be infinite - it is a 2-D problem - and $d/W << 1$) is hinged at o ne edge in contact with the bottom of a tank filled with a viscous liquid. The angle sepa rating the plate and the plane is given by α. If we release the plate it will settle toward t he plane (e.g., α will decrease in time). In the limit $\alpha<<1$, develop an equation governin g α as a function of time.



Hint: The net torque on the plate will be zero - the torque due to lubrication forces will balance the torque due to gravity. You may find the r momentum equation useful:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Problem 4). The drag on a particle under creeping flow conditions is characterized by a *resistance tensor* R_{ij}:

$$F_i = R_{ij} U_j$$

In this problem we consider the hydrodynamics of a prolate spheroid with major axis a and minor axis b (the shape you get if you rotate an ellipse around its major axis - essen tially a football shape with round ends). The orientation of the spheroid is determined by the orientation vector p_i of its major axis.

a. Determine the most general form for the resistance tensor.

b. Using the minimum dissipation theorem and Stokes Law (and any physical intuition you may have), determine upper and lower bounds for the coefficients.