1. A classic problem in Taylor-Aris dispersion is the spreading of peaks in chromatography. The separation between peaks arises because one species is selectively absorbed into a stationary phase, and thus has a smaller average velocity than a species which isn’t absorbed. The spreading occurs because it takes time for the molecules to diffuse into and out of the stationary phase, as well as diffuse across the mobile phase. In this problem we look at a very simplified example of this effect.

Consider the channel of width 2b in the y direction as depicted below. We shall have a stationary phase of thickness δ at each wall (this would be analogous to the technique of open channel liquid chromatography). To make your calculations easier, we shall assume that the partition coefficient between the two phases is unity (this would not in general be the case!) and further that the diffusion coefficient in the mobile and stationary phases is the same (also not normally the case!). Finally, we shall assume that in the mobile phase region we have a plug flow velocity \( U_m \) (rather than a parabolic profile) and in the stationary phase region we have a velocity of zero. Our velocity profile is thus:

\[
0 < y < b-\delta: \quad u = U_m \\
\delta < y < b: \quad u = 0
\]

and thus the average velocity \( U \) is just \( U_m (1-\delta/b) \).

So: Using the method of moments, calculate the Taylor dispersivity as a function of \( \delta/b \).

Hint: The problem as proposed is essentially the same as the parabolic channel flow problem solved in the notes, the only difference being the velocity profile (which is now discontinuous at \( y = b-\delta \)). When solving for \( c_i \) (the only hard part!) you will need to break it into two regions and solve for \( f \) in each, with the usual continuous value and derivative matching conditions at the interface.

2. A porous sphere of radius \( a \) is a source of fluid at a total flow rate \( Q \) as depicted below. The sphere is maintained at a temperature \( T_1 \) (as is the fluid emanating from the surface) and the temperature far away is at temperature \( T_0 \).

a). Write down the solution for the velocity profile. This is simple, spherically symmetric source flow. Everything is only a function of \( r \), and the velocity is only in the radial direction!
b). Write down the equation governing energy transport and render it dimensionless. Show that the problem is a function of a single dimensionless parameter.

c). (This is the important part) **Prove** that the problem will admit a regular perturbation expansion for small Peclet numbers. Obtain the first two terms of the expansion.