Name: ______________________

Instructor: ______________________

Exam 1, Math 20580
Feb. 15, 2024

- The Honor Code *is* in affect for this examination. All work is to be your own.
- Please turn off and stow all cellphones and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 75 minutes.
- Be sure that your name and your instructor's name and your section number are on the front page of your exam.
- There are 12 problems, 8 are multiple choice and 4 are partial credit.
- Be sure that you have all 8 pages of this exam.
- Multiple choice questions should have distinct answers. (If for some reason you think this is not the case, let your instructor know *after the exam* and do one of the following. If you think a multiple choice question has no listed correct answer, leave the line blank. If you are right you will get full credit. If you think a multiple choice question has more than one correct answer, X ONE of them: you will never get credit for a line with two or more X's.)

Please mark you answers with an **X**, not a circle.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
			1		
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
			2		
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
			3		
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
			4		

MC Total.	
9	
10.	
11.	
12.	
Total.	

Multiple choice problems

1. (7 points) If A and B are 2×2 matrices such that $A = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$, then what is the matrix B?

(a) $\begin{bmatrix} 4 & 6 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 2 \\ 6 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

2. (7 points) Let A be the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & -1 & 2 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Which of the following matrices is the reduced row echelon form (RREF) of A?

(a) $\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

3. (7 points) For which of the following equations are there 2×2 matrices A and B which do NOT satisfy the equation:

$$\text{(I):}\quad A(A+B) = A^2 + AB \qquad \text{(II):}\quad A^2 - B^2 = (A-B)(A+B)$$
$$\text{(III):}\quad AB = BA \qquad \text{(IV):}\quad B(A+B) = BA + B^2.$$

(a) (II) only

(b) (I), (III) and (IV) only

(c) (I), (II) and (IV)

(d) (I) and (IV) only

(e) (II) and (III) only

4. (7 points) For which values of h and k is the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & h & k & 1 \end{bmatrix}$ in reduced row echelon form (RREF)?

(a) $h = 1$ and any k

(b) $(h, k) = (0, 0)$ only

(c) $(h, k) = (0, 0)$ and $(h, k) = (0, 1)$ only.

(d) $(h, k) = (1, 0)$ and $(h, k) = (1, 1)$ only

(e) $h = 0$ and any k

5. (7 points) Let A be a 5×6 matrix such that the dimension of the null space of A is 4. What is the dimension of the null space of the transpose matrix A^T?

(a) 3 (b) 2 (c) 1 (d) 0 (e) 4

6. (7 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by counterclockwise rotation of the plane about the origin by angle $\frac{3\pi}{2}$ (in radians). Which of the following is the standard matrix of T?

(a) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

7. (7 points) Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{v}_4 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

in $\mathbb{R}^3$. Which of the following statements is true?

(a) $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent

(b) $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ spans $\mathbb{R}^3$

(c) $\{\vec{v}_1, \vec{v}_2, \vec{v}_4\}$ is linearly independent

(d) $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is a basis of $\mathbb{R}^3$

(e) $\{\vec{v}_1, \vec{v}_2\}$ is linearly dependent

8. (7 points) Consider the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

for $\mathbb{R}^3$. Let $\vec{x}$ denote the vector

$$\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \quad \text{and write} \quad [x]_{\mathcal{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

for the coordinate vector of $\vec{x}$ with respect to $\mathcal{B}$. Find a, b and c. Which number below is the value of c?

(a) 3 (b) -5 (c) -1 (d) 5 (e) 1

Partial credit problems

9. (11 points) The matrix A has the reduced row echelon form (RREF) B, as shown below:

$$A = \begin{bmatrix} 0 & 0 & 3 & -3 & 1 \\ 1 & -1 & -4 & 7 & 7 \\ -2 & 2 & 2 & -8 & -5 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis of the column space col(A) of A.

(b) Find a basis of the null space null(A) of A.

10. (11 points) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 1 & 3 \end{bmatrix}.$$

11. (11 points)

(a) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by the formula

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_3 \\ x_1 \\ x_3 \end{bmatrix}.$$

Write down the standard matrix $A = [T]$ of T.

(b) Let $S\colon \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by the formula $S(\vec{x}) = B\vec{x}$ for $\vec{x}$ in $\mathbb{R}^4$ where

$$B = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

Fill in the ... below to give a formula for S like that given for T in (a):

$$S\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} \cdots\cdots\cdots \\ \cdots\cdots\cdots \end{bmatrix}$$

(c) Compute explicitly the standard matrix $C = [S \circ T]$ of the composite $S \circ T$ of T and S.

12. (11 points) Consider the two bases $\mathcal{B}$ and $\mathcal{C}$ of $\mathbb{R}^2$ given by

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}, \qquad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}.$$

(a) Let $\vec{x}$ be the vector in $\mathbb{R}^2$ with $\mathcal{B}$-coordinate vector

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}.$$

Find $\vec{x}$ explicitly as a vector in $\mathbb{R}^2$ i.e. write $\vec{x}$ in the form $\vec{x} = \begin{bmatrix} a \\ b \end{bmatrix}$ for some scalars a and b.

(b) Find the change of coordinates matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ from $\mathcal{B}$ to $\mathcal{C}$ (recall that $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ is the matrix such that $[\vec{v}]_{\mathcal{C}} = \underset{\mathcal{C}\leftarrow\mathcal{B}}{P} \cdot [\vec{v}]_{\mathcal{B}}$ for all vectors $\vec{v}$ in $\mathbb{R}^2$).

(c) Use (b) to find the $\mathcal{C}$-coordinate vector $[\vec{x}]_{\mathcal{C}}$ of the vector $\vec{x}$ in (a).