Name: ______________________

Instructor: ______________________

Exam 1, Math 20580
Feb. 15, 2024

- The Honor Code *is* in affect for this examination. All work is to be your own.
- Please turn off and stow all cellphones and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 75 minutes.
- Be sure that your name and your instructor's name and your section number are on the front page of your exam.
- There are 12 problems, 8 are multiple choice and 4 are partial credit.
- Be sure that you have all 8 pages of this exam.
- Multiple choice questions should have distinct answers. (If for some reason you think this is not the case, let your instructor know *after the exam* and do one of the following. If you think a multiple choice question has no listed correct answer, leave the line blank. If you are right you will get full credit. If you think a multiple choice question has more than one correct answer, X ONE of them: you will never get credit for a line with two or more X's.)

Please mark you answers with an **X**, not a circle.

1.	(a)	(b)	(c) X	(d)	(e)
2.	(a)	(b)	(c)	(d) X	(e)
			1		
3.	(a)	(b)	(c)	(d)	(e) X
4.	(a)	(b)	(c) X	(d)	(e)
			2		
5.	(a) X	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e) X
			3		
7.	(a)	(b)	(c) X	(d)	(e)
8.	(a)	(b)	(c)	(d) X	(e)
			4		

MC Total.	
9	
10.	
11.	
12.	
Total.	

Multiple choice problems

1. (7 points) If A and B are 2×2 matrices such that $A = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$, then what is the matrix B?

(a) $\begin{bmatrix} 4 & 6 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 2 \\ 6 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$$B = A^{-1}(AB) = \frac{1}{1\times 3 - (-1)\cdot(-1)} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & 6 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

2. (7 points) Let A be the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & -1 & 2 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Which of the following matrices is the reduced row echelon form (RREF) of A?

(a) $\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 & -1 & 2 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[R3 \to R3 - R1]{R2 \to R2 - R1} \begin{bmatrix} 1 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\xrightarrow[R2 \to R2 - R3]{R1 \to R1 + R3} \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

3. (7 points) For which of the following equations are there 2×2 matrices A and B which do NOT satisfy the equation:

$$\text{(I)}: \quad A(A+B) = A^2 + AB \qquad \text{(II)}: \quad A^2 - B^2 = (A-B)(A+B)$$
$$\text{(III)}: \quad AB = BA \qquad \text{(IV)}: \quad B(A+B) = BA + B^2.$$

(a) (II) only (b) (I), (III) and (IV) only (c) (I), (II) and (IV)

(d) (I) and (IV) only (e) (II) and (III) only

(I) and (IV) follow from associativity of matrix multiplication.
(III) is false (matrix multiplication is not commutative)
The only option which includes (III) but neither (I) nor (IV) is (e); this must be the answer.
To see why (II) is false, note by associativity,

$$\begin{aligned}(A-B)(A+B) &= A(A+B) - B(A+B) \\ &= A^2 + AB - (BA + B^2) \\ &= A^2 - B^2 + (AB - BA).\end{aligned}$$

So $(A-B)(A+B) = A^2 - B^2 \iff AB = BA$ which is false in general for 2×2 matrices, as already noted.

4. (7 points) For which values of h and k is the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & h & k & 1 \end{bmatrix}$ in reduced row echelon form (RREF)?

(a) $h = 1$ and any k (b) $(h,k) = (0,0)$ only

(c) $(h,k) = (0,0)$ and $(h,k) = (0,1)$ only. (d) $(h,k) = (1,0)$ and $(h,k) = (1,1)$ only

(e) $h = 0$ and any k

h can't be a leading entry since the 2 above it is non-zero. So h must be zero. Once we know $h=0$, if k is the leading entry of row 2, it must be 1, giving the RREF $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. If $h=0$ and k is not the leading entry of row 2, it must be 0, giving the RREF $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. So $(h,k) = (0,0)$ and $(h,k) = (0,1)$ are the only possibilites.

5. (7 points) Let A be a 5×6 matrix such that the dimension of the null space of A is 4. What is the dimension of the null space of the transpose matrix A^T?

(a) 3 (b) 2 (c) 1 (d) 0 (e) 4

$$\text{rank}(A) + \underbrace{\dim \text{null}(A)}_{4} = \#(\text{cols of } A) = 6$$

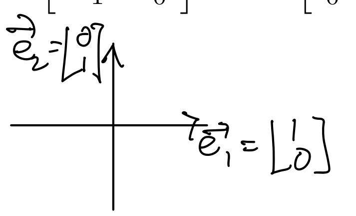
So $\text{rank}(A) = 2$. $\text{rank}(A^T) = \text{rank}(A) = 2$.

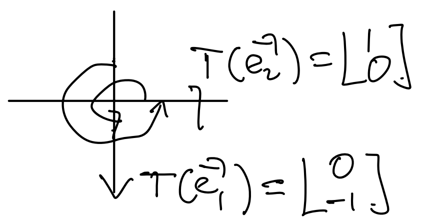
$$\underbrace{\text{rank}(A^T)}_{2} + \dim \text{null}(A^T) = \#(\text{cols of } A^T) = 5$$

so $\dim \text{null}(A^T) = 3$

6. (7 points) Let $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by counterclockwise rotation of the plane about the origin by angle $\frac{3\pi}{2}$ (in radians). Which of the following is the standard matrix of T?

(a) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$





$$[T(\vec{e_1}) \; T(\vec{e_2})] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

7. (7 points) Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{v}_4 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

in $\mathbb{R}^3$. Which of the following statements is true?

(a) $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent

(b) $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ spans $\mathbb{R}^3$

(c) $\{\vec{v}_1, \vec{v}_2, \vec{v}_4\}$ is linearly independent

(d) $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is a basis of $\mathbb{R}^3$

(e) $\{\vec{v}_1, \vec{v}_2\}$ is linearly dependent

$\vec{v}_3 = \vec{v}_2 - \vec{v}_1$ so (a) is false.

If (b) is true, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis of $\mathbb{R}^3$ (since $\dim \mathbb{R}^3 = 3$) so (a) would be true. So (b) is false.

(d) is false since a basis of $\mathbb{R}^3$ has 3 vectors

(e) is false since $\vec{v}_1 \neq 0$ and $\vec{v}_2$ is not a scalar multiple of $\vec{v}_1$.

This leaves (c) as the only possible correct answer. To check (c) is true, row-reduce

$$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -1 \\ 0 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 2 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \text{ (REF).}$$

There are three pivots, so $\vec{v}_1, \vec{v}_2$ and $\vec{v}_4$ are linearly independent.

8. (7 points) Consider the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

for $\mathbb{R}^3$. Let $\vec{x}$ denote the vector

$$\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \quad \text{and write} \quad [x]_{\mathcal{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

for the coordinate vector of $\vec{x}$ with respect to $\mathcal{B}$. Find a, b and c. Which number below is the value of c?

(a) 3 (b) -5 (c) -1 (d) 5 (e) 1

$$[a]_{\mathcal{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ means } \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} = a\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a+b+c \\ a+b \\ a \end{bmatrix}$$

So $a=5$, $a+b=-2$ and $b=-7$, $a+b+c=3$ and $c=5$.

Partial credit problems

9. (11 points) The matrix A has the reduced row echelon form (RREF) B, as shown below:

$$A = \begin{bmatrix} 0 & 0 & 3 & -3 & 1 \\ 1 & -1 & -4 & 7 & 7 \\ -2 & 2 & 2 & -8 & -5 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis of the column space col(A) of A.

The 1st, 3rd, 5th columns of B contain pivots, so the 1st, 3rd, 5th columns of A are a basis of col(A). So col(A) has basis

$$\begin{bmatrix} 0 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ -5 \\ 2 \end{bmatrix}$$

(b) Find a basis of the null space null(A) of A.

$A\vec{x} = \vec{0}$ is equivalent to $B\vec{x} = \vec{0}$, or, writing $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$, to

$$\begin{cases} x_1 - x_2 + 3x_4 = 0 \\ x_3 - x_4 = 0 \\ x_5 = 0. \end{cases}$$

The free variables are x_2 and x_4, corresponding to non-pivot columns of B. Moving the free variables to the right gives

$$\begin{cases} x_1 = x_2 - 3x_4 \\ x_3 = x_4 \\ x_5 = 0 \end{cases}$$

or, writing $x_2 = r$ and $x_4 = s$,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} r - 3s \\ r \\ s \\ s \\ 0 \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

The vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ form a basis of null(A).

10. (11 points) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 1 & 3 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \longrightarrow \begin{bmatrix} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & 1 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & +2 & -1 & -1 \end{bmatrix} \xleftarrow{2} \begin{bmatrix} 1 & 1 & 0 & -3 & 2 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{bmatrix}$$

So $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} -3 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & -1 & -1 \end{bmatrix}.$

Check: $\begin{bmatrix} -3 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. (11 points)

(a) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation given by the formula

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_3 \\ x_1 \\ x_3 \end{bmatrix}.$$

Write down the standard matrix $A = [T]$ of T.

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix},\ T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix},\ T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \text{ so}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Let $S\colon \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transformation given by the formula $S(\vec{x}) = B\vec{x}$ for $\vec{x}$ in $\mathbb{R}^4$ where

$$B = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

Fill in the ... below to give a formula for S like that given for T in (a):

$$S\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_3 \\ x_4 - x_2 \end{bmatrix}$$

If $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, $B\vec{x} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 - x_3 \\ x_4 - x_2 \end{bmatrix}$

(c) Compute explicitly the standard matrix $C = [S \circ T]$ of the composite $S \circ T$ of T and S.

$$C = [S][T] = BA = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

12. (11 points) Consider the two bases $\mathcal{B}$ and $\mathcal{C}$ of $\mathbb{R}^2$ given by

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}, \qquad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}.$$

(a) Let $\vec{x}$ be the vector in $\mathbb{R}^2$ with $\mathcal{B}$-coordinate vector

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}.$$

Find $\vec{x}$ explicitly as a vector in $\mathbb{R}^2$ i.e. write $\vec{x}$ in the form $\vec{x} = \begin{bmatrix} a \\ b \end{bmatrix}$ for some scalars a and b.

$$\vec{x} = -6\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ -13 \end{bmatrix}$$

or $\vec{x} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -6 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -13 \end{bmatrix}$.

(b) Find the change of coordinates matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ from $\mathcal{B}$ to $\mathcal{C}$ (recall that $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ is the matrix such that $[\vec{v}]_{\mathcal{C}} = \underset{\mathcal{C}\leftarrow\mathcal{B}}{P} \cdot [\vec{v}]_{\mathcal{B}}$ for all vectors $\vec{v}$ in $\mathbb{R}^2$).

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 2 \\ 2 & 5 & 2 & -1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 3 & 1 & 2 \\ 0 & -1 & 0 & -5 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 1 & -13 \\ 0 & 1 & 0 & 5 \end{array}\right]$$

$$\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 1 & -13 \\ 0 & 5 \end{bmatrix}$$

or $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \underset{\mathcal{C}\leftarrow\mathcal{E}}{P}\,\underset{\mathcal{E}\leftarrow\mathcal{B}}{P} = \left(\underset{\mathcal{E}\leftarrow\mathcal{C}}{P}\right)^{-1}\left(\underset{\mathcal{E}\leftarrow\mathcal{B}}{P}\right)$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \frac{1}{1\times 5 - 2\times 3}\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= -\begin{bmatrix} -1 & 13 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 5 \end{bmatrix}$$

(c) Use (b) to find the $\mathcal{C}$-coordinate vector $[\vec{x}]_{\mathcal{C}}$ of the vector $\vec{x}$ in (a).

$$[\vec{x}]_{\mathcal{C}} = \underset{\mathcal{C}\leftarrow\mathcal{B}}{P}[x]_{\mathcal{B}} = \begin{bmatrix} 1 & -13 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} -6 \\ 1 \end{bmatrix} = \begin{bmatrix} -19 \\ 5 \end{bmatrix}$$