**Math 20580** Name:____________________
**Midterm 3** Instructor:____________________
**November 14, 2017** Section:____________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an × through one letter for each problem on this answer sheet.

**Sign the pledge.** "On my honor, I have neither given nor received unauthorized aid on this Exam":

________________________________________

1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e

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Multiple Choice.

9.

10.

11.

12.

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Total.

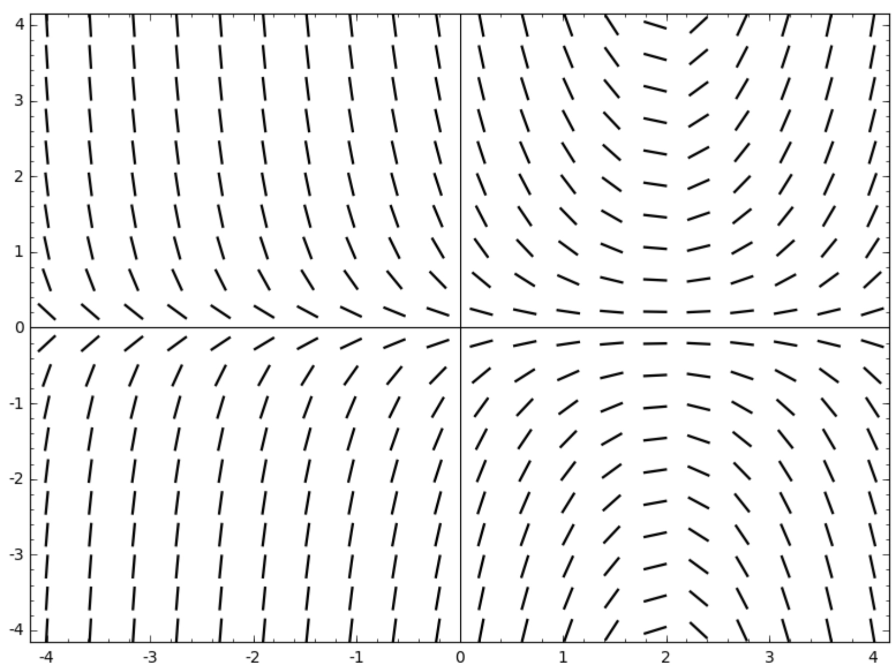
## Part I: Multiple choice questions (7 points each)

1. Which of the following functions is a solution of the initial value problem

$$(y' - \sin x)^2 = 1 + x^2 - y^2, \quad y(0) = 1$$

(a) $-\sin x$ (b) $x \sin x + \cos x$ (c) $\cos x$ (d) $x \cos x - \sin x$ (e) $\sin x - \cos x$

2. Determine $f(t, y)$ if the differential equation $y' = f(t, y)$ has direction field (the value of $t$ is measured on the horizontal axis, and the value of $y$ on the vertical axis)




(a) $\sin(t) + y$ (b) $y + t^2$ (c) $t \sin(y)$ (d) $ty - 2y$ (e) $e^y(t - 1)$

3. Consider the orthogonal vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$ and let $V = \text{Span}\{\vec{v}_1, \vec{v}_2\}$. The matrix of the projection onto $V$ is

(a) $\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 1/9 & -2/9 & 0 \\ 2/9 & 2/9 & 0 \\ 2/9 & -1/9 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 5/9 & -2/9 & 4/9 \\ -2/9 & 8/9 & 2/9 \\ 4/9 & 2/9 & 5/9 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & 2 \\ -2 & 2 & 1 \end{bmatrix}$

4. Let $A$ be an $m \times n$ matrix with linearly independent columns and let $\vec{b}$ in $\mathbb{R}^m$ be a vector which is not in $Col(A)$. Which of the following statements may be false?

(a) There exists a vector $\vec{x}$ in $\mathbb{R}^n$ with $A\vec{x} - \vec{b}$ perpendicular to $Col(A)$.

(b) $\det(A^T A) \neq 0$.

(c) $m > n$.

(d) The vector $\vec{b}$ is not the zero vector.

(e) $\det(AA^T) \neq 0$.

5. Find the solution to the initial value problem

$$t\frac{dy}{dt} + 3y = \frac{t}{1+t^4}, \qquad y(1) = 0.$$

(a) $y = \ln\left(\frac{1+t^4}{2t^3}\right)$ (b) $y = t^3 - 1$ (c) $y = \frac{1}{4t^3} \cdot \ln\left(\frac{1+t^4}{2}\right)$

(d) $y = \frac{1}{2} \cdot \arctan(t^2) - \pi/8$ (e) $y = \frac{4t^3 - 4}{1+t^4}$

6. Which of the following functions can be used as an integrating factor for the equation $y' + ty = \cos t$?

(a) $t$ (b) $e^{t^2/2}$ (c) $t^2/2$ (d) $e^t$ (e) $e^{\cos t}$

7. The ordinary differential equation

$$(2xy^2 + 2y) + (2x^2y + 2x)y' = 0$$

is

(a) linear (b) autonomous (c) separable (d) an equation of order 2
(e) none of the above.

8. The solution of the initial value problem

$$x \cdot \frac{dy}{dx} = y + xy, \qquad y(1) = 2$$

is the function

(a) $y = \dfrac{e^x}{2(x+1)}$ (b) $y = \ln(x) + 2x$ (c) $y = x^2 + x$ (d) $y = 2$ (e) $y = 2xe^{x-1}$.

**Part II: Partial credit questions (11 points each). Show your work.**

9. Let $W = \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, where

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 2 \\ -4 \\ 4 \\ -2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -3 \end{bmatrix}.$$

(a) Apply the Gram-Schmidt process to find an orthonormal basis for $W$.

(b) Find the $QR$ decomposition of the matrix $A$ with columns $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

10. Let $A = \begin{bmatrix} -1 & 1 \\ -6 & 4 \\ 2 & -1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$.

(a) Find the least squares solution to the equation $A\vec{x} = \vec{b}$.

(b) Find the vector in the column space of $A$ which is closest to $\vec{b}$.

11. A tank initially contains 50 liters of water and 20 grams of salt. Water containing a salt concentration of 2 g/L enters the tank at the rate of 5 L/min, and the well-stirred mixture leaves the tank *at the same rate*.

(a) Find an expression for the amount of salt in the tank at any time $t$.

(b) How long does it take for the amount of salt to reach 60 grams.

(c) Find the approximate amount of salt after 100 years.

12. (a) Find, in terms of $y_0$, the solution of the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = t \cdot e^y \\ y(0) = y_0 \end{cases}$$

(b) Find the maximal interval on which the solution to the initial value problem above exists, and explain how this interval depends on $y_0$.