

M20580 L.A. and D.E. Tutorial
Quiz 2

1. From the multiple choice answers listed below, select the proper rank of the matrix given.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

A. $\text{rank}(A)=3$ B. $\text{rank}(A)=2$ C. $\text{rank}(A)=1$ D. $\text{rank}(A)=0$

A is in REF and there are 2 pivot variables $\Rightarrow \text{rank}(A)=2$

2. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a transformation such that:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} |x| \\ |y| \end{bmatrix}$$

where $|x|$ and $|y|$ indicate the absolute values of x and y , respectively. Either prove that it is a linear transformation or give a counterexample for why it is not.

There are many counterexamples. Here are 2:

$$i) T\left(\begin{bmatrix} -1 \cdot x \\ -1 \cdot y \end{bmatrix}\right) = \begin{bmatrix} |-1 \cdot x| \\ |-1 \cdot y| \end{bmatrix} = \begin{bmatrix} |x| \\ |y| \end{bmatrix} \neq -I \begin{bmatrix} |x| \\ |y| \end{bmatrix} = -I \cdot T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$$

$$ii) T\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} |-1+1| \\ |-1+1| \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} |-1| \\ |-1| \end{bmatrix} + \begin{bmatrix} |1| \\ |1| \end{bmatrix} \\ = T\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix}\right) + T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$$