**M20580 L.A. and D.E. Tutorial**
**Quiz 3**

*3'*

*linear dependent*

1. Consider the following vectors.

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$

Which of the following sets of vectors is linearly independent?

(a) $\{\ \vec{u},\ \vec{w}\ \}$

(b) $\{\ \vec{u},\ \vec{v},\ \vec{x}\ \}$

(c) $\{\ \vec{v},\ \vec{x}\ \}$

(d) $\{\ \vec{u},\ \vec{v}\ \}$

(e) $\{\ \vec{u},\ \vec{v},\ \vec{w},\ \vec{x}\ \}$

*7'*

2. Consider the matrix $A$ and vector $\mathbf{v}$ as follows:

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 4 \\ 6 \\ -4 \\ 2 \end{bmatrix}.$$

Find a basis $\mathcal{B}$ for Col(A). Is vector $\mathbf{v}$ in Col(A)?

$$\text{REF of } A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \implies \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 4 \\ 2 & 1 & 1 & | & 6 \\ -1 & -1 & 1 & | & -4 \\ 1 & 0 & 2 & | & 2 \end{bmatrix} \xrightarrow{\text{REF}} \begin{bmatrix} 1 & 0 & 0 & | & 4 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & -1 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}. \quad \text{Yes.}$$