Name: Date: 2/22/2024

M20580 L.A. and D.E. Tutorial
Quiz 4

1. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation whose matrix in standard coordinates is $[T]_E = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$. Consider the following vectors in standard coordinates.

$$\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}.$$

Which of the following vectors is in the kernel of T?

(a) $\vec{v} + \vec{x}$

(b) $\vec{u} - \vec{v}$

(c) $\vec{w} + \vec{x}$

(d) $\vec{w} - \vec{v}$

(e) $\vec{u} + \vec{v}$

$T(u) = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $T(v) = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$,

$T(w) = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$, $T(x) = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = T(u) - T(v) = T(u-v)$

2. Determine whether the set $\{1-x, 1-x^2, x-x^2\}$ is a basis for $\mathcal{P}_2$.

$a(1-x) + b(1-x^2) + c(x-x^2)$

$= a - ax + b - bx^2 + cx - cx^2$

$= (a+b) + (-a+c)x + (-b-c)x^2 = 0$ Check L.I.

$$\Longleftrightarrow \begin{aligned} a+b \quad &= 0 \\ -a \quad +c &= 0 \\ -b-c &= 0 \end{aligned} \Longleftrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right) \xrightarrow{R_2+R_1 \mapsto R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right) \xrightarrow{R_3+R_2 \mapsto R_3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1-R_2 \mapsto R_1} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

so $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ is nontrivial solution

Corresponds to $a = s$, $b = -s$, $c = s$ for free variable s. So this set is not L.I. and hence not a basis.