

**M20580 L.A. and D.E.**  
**Quiz 5**

1. Find the determinant of the following  $4 \times 4$  matrix by expanding along the third row:

$$\begin{bmatrix} 0 & 2 & -1 & 1 \\ 4 & 3 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$

**Solution:** Performing a Laplace expansion on the third row we see that our  $4 \times 4$  determinant is equal to  $-2 \times \det \left( \begin{bmatrix} 0 & -1 & 1 \\ 4 & 1 & 1 \\ 3 & 1 & 0 \end{bmatrix} \right) = (-2)(-2) = 4$ .

2. Use Cramer's rule to solve the system

$$\begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(a)  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0.5 \end{bmatrix}$       (b)  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}$       (c)  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(d)  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$       (e)  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix}$

**Solution:** Answer choice (b) is correct.

The determinant of the matrix is 2. Replacing the first column with the solution vector we obtain the matrix  $\begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$  which has determinant 3. Hence  $x_1 = 3/2$ .

Replacing the second column with the solution vector we obtain the matrix  $\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$  which has determinant 1. Hence  $x_2 = 1/2$ .