Name: Date: 03/21/2024

Math 20580 L.A. and D.E. Tutorial
Quiz 6

1. Let $V = M_{2\times 3}$ be the vector space of 2×3 matrices and consider the subspace

$$W = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} : a + b + c = 0 \text{ and } d = 0 \right\}.$$

a.) Find a basis for W.

b.) What is the dimension of W?

Solution: Substituting the relations $a = -b - c$ and $d = 0$ into a typical element of $M_{2\times 3}$, we see that a typical element of W is given by

$$\begin{bmatrix} -b-c & b & c \\ 0 & e & f \end{bmatrix} = b \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + c \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + e \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + f \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

We see that $W = \operatorname{span}\left\{ \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$. Therefore, a basis for W is $\left\{ \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$. Since there are four elements in this basis for W, then the dimension of W is 4.

2. Consider the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$. Is $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ an eigenvector of A? If so, what is its associated eigenvalue?

(a) Yes; 7 **(b)** Yes; -4 **(c)** $\mathbf{x}$ is not an eigenvector **(d)** Yes; 0 **(e)** Yes; 4

Solution: Answer choice (e) is correct (**$\mathbf{x}$ is an eigenvector corresponding to eigenvalue 4**). We know that, if $\mathbf{x}$ is an eigenvector, it should satisfy the equation

$$A\mathbf{x} = \lambda \mathbf{x}$$

for some number λ. Now, we observe that

$$A\mathbf{x} = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix} = 4\mathbf{x}$$

so $\mathbf{x}$ is an eigenvector of A corresponding to eigenvalue 4.