M20580 L.A. and D.E. Tutorial
Quiz 7

1. Let A be 3×3 matrix. Suppose that A is diagonalizable and the eigenvalues of A are $-1, 0, 1$ with eigenvectors

$$v_{-1} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, v_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

respectively. Find A^8.

Hint: Use $P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $Q = P^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

> **Solution:** Since A is diagonalizable, then
>
> $$A = P \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} P^{-1}$$
>
> and
>
> $$A^8 = P \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} P^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}.$$

2. Let $W = \text{span}\{ \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} \}$. Which of the following vectors is in $W^\perp$?

(a) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

> **Solution:** Since the vector is in $W^\perp$ if and only if it is orthogonal to both $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$,
>
> only (e) is possible.