Name: Date: 04/04/2024

**M20580 L.A. and D.E. Tutorial**
**Quiz 8**

1. Let $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ be a basis of $\mathbb{R}^3$. Using the Gram-Schmidt process to find an associated orthogonal basis, the first two orthogonal vectors are:

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

What is the third?

A. $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ B. $\begin{bmatrix} -1/3 \\ 1/3 \\ 2/3 \end{bmatrix}$ C. $\begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \end{bmatrix}$ D. $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ E. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

B is the answer. You can immediately see this's as it is the only option orthogonal to both vectors.
Letting $\vec{x}_3$ be the third basis vector and $\vec{v}_1, \vec{v}_2$ be the first two orthogonal vectors, we can also directly compute

$$\vec{v}_3 = \vec{x}_3 - \frac{\vec{v}_1 \cdot \vec{x}_3}{\|\vec{v}_1\|^2}\vec{v}_1 - \frac{\vec{v}_2 \cdot \vec{x}_3}{\|\vec{v}_2\|^2}\vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \frac{0}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 1/3 \\ 2/3 \end{bmatrix}$$

2. Find the least squares solution to $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

(Hint: $(A^TA)^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$)

$$\vec{x} = (A^TA)^{-1}A^T\vec{b} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$