Name: ____________________

Section #: ____________________

Exam 2, Math 20580
March 6, 2025

- The Honor Code *is* in affect for this examination. All work is to be your own.
- Please turn off and put away all cellphones and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name and your instructor's name and your section number are on the front page of your exam.
- There are 12 problems: 8 are multiple choice and 4 are partial credit.
- Be sure that you have all 8 pages of this exam.
- Each multiple choice question is worth 6 points. Your score will be the sum of the best 7 scores on the multiple choice plus your score on questions 9–12.
- Make sure you have correctly marked your multiple choice answers by the time the exam ends. No credit will be given for incorrectly marked answers.

01	Matthew Dyer	8:20-9:10	127 HH
02	Matthew Dyer	9:25-10:15	127 HH
03	Jeffrey Diller	10:30-11:20	136 DBRT
04	Han Lu	11:30-12:20	114 PASQ
05	Sudipta Gosh	12:50-1:40	140 DBRT
06	Sudipta Gosh	2:00-2:50	140 DBRT
07	Claudia Polini	3:30-4:20	140 DBRT

Please mark you answers with an **X**, not a circle.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
			1		
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
			2		
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
			3		
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
			4		

MC Total.	
9.	
10	
11.	
12.	
Total.	

1. (6 points) Which of the following is a vector space?

I. $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| x, y, z \text{ in } \mathbb{R} \text{ and } x + y + z = 1 \right\}$.

II. $\left\{ \begin{bmatrix} 0 \\ x \\ y \end{bmatrix} \middle| x, y \text{ in } \mathbb{R} \text{ and } y = x \text{ or } y = -x \right\}$.

III. $\left\{ \begin{bmatrix} t \\ t+s \\ s-t \end{bmatrix} \middle| s, t \text{ in } \mathbb{R} \right\}$.

IV. $\left\{ \begin{bmatrix} s \\ t \\ 0 \end{bmatrix} \middle| s, t \text{ in } \mathbb{R} \text{ and } t \geq 0 \right\}$.

(a) III and IV. (b) IV only. (c) II, III and IV. (d) I and III. (e) III only.

2. (6 points) Let $M_{2,2}$ denote the set of all 2×2 matrices, and let S be the subspace of $M_{2,2}$ consisting of all matrices A satisfying $A + A^T = 0$. What is the dimension of S?

(a) 2 (b) 3 (c) 1 (d) 0 (e) 4

3. (6 points) Let $\mathcal{P}_2$ denote the vector space of polynomials of degree at most 2, and let $T\colon \mathcal{P}_2 \to \mathcal{P}_2$ be a linear transformation such that

$$T(1+x) = x, \quad T(x+x^2) = x^2 \quad \text{and} \quad T(x^2+1) = 1.$$

What is the value of $T(2+2x+2x^2)$?

(a) $1+x+x^2$ (b) 2 (c) $4x+x^2$ (d) $4+4x+4x^2$ (e) $x+x^2$

4. (6 points) Let $\mathcal{P}_3$ denote the vector space of polynomials of degree at most 3. Let $T\colon \mathbb{R}^3 \to \mathcal{P}_3$ be a linear transformation. Which of the following statements is always correct?

I. T can not be one-to-one.
II. T can not be onto.
III. If T is one-to-one, then the dimension of range(T) is 3.
IV. $\text{rank}(T) + \text{nullity}(T) = 4$.

(a) II, III and IV. (b) II and IV. (c) I only. (d) III only. (e) II and III.

5. (6 points) Let A, B and C be 2×2 matrices such that $\det(A) = 3$, $\det(B) = \sqrt{2}$ and C is invertible. What is the value of $\det(3C^{-1}AB^2C^T)$, where C^T denotes the transpose of C?

(a) 108 (b) 18 (c) 54 (d) 27 (e) 6

6. (6 points) Let $\mathcal{P}_1$ denote the vector space of polynomials of degree at most 1. Consider the two bases $\mathcal{E} = \{1, x\}$ and $\mathcal{B} = \{1 - 2x, -1 + x\}$ of $\mathcal{P}_1$. Which of the following is the first row of the change of basis matrix $\underset{\mathcal{B} \leftarrow \mathcal{E}}{P}$ from $\mathcal{E}$ to $\mathcal{B}$?

(a) $[-1 \quad -2]$ (b) $[-2 \quad -1]$ (c) $[1 \quad 1]$ (d) $[-1 \quad -1]$ (e) $[1 \quad -2]$

7. (6 points) Find the area of the parallelogram with vertices $(0,0)$, $(6,7)$, $(4,5)$ and $(10,12)$.

(a) 3 (b) 1 (c) 4 (d) 2 (e) 5

8. (6 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 9 \end{bmatrix}.$$

What is the determinant of A?

(a) 0 (b) -4 (c) 4 (d) 3 (e) -3

9. (15 points) Consider the bases of $\mathcal{P}_2$ given by

$$\mathcal{B} = \{4 + 3x + x^2, 1 + x - 2x^2, 1 - x^2\} \quad \text{and} \quad \mathcal{C} = \{1 + x, 1 + x^2, 1 - x^2\}.$$

(a) Find the change of basis matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ from $\mathcal{B}$ to $\mathcal{C}$.

(b) Suppose that $p(x)$ is a polynomial in $\mathcal{P}_2$ with $[\,p(x)]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$. What is $[\,p(x)]_{\mathcal{C}}$?

(c) What is $p(x)$ in (b)?

10. (14 points) Consider the matrix

$$A = \begin{bmatrix} s & -1 & 0 \\ -1 & s & -1 \\ 0 & -2 & s \end{bmatrix}$$

where s is a real number.

(a) Compute the determinant of A.

(b) For which values of s is the matrix A invertible?

(c) If the matrix A is invertible, give a formula for $\det(A^{-1})$ in terms of s.

11. (15 points) Let $\mathcal{P}_3$ denote the vector space of polynomials of degree at most 3, and $T\colon \mathcal{P}_3 \to \mathbb{R}^2$ be the linear transformation given by

$$T(p(x)) = \begin{bmatrix} p(-1) \\ p(1) \end{bmatrix}$$

for $p(x)$ in $\mathcal{P}_2$ (you do not have to explain why T is a linear transformation).

(a) Write down the matrix $\underset{\mathcal{E}\leftarrow\mathcal{B}}{[T]}$ of T with respect to the standard bases

$$\mathcal{B} = \{1, x, x^2, x^3\} \text{ of } \mathcal{P}_3 \quad \text{and} \quad \mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \text{ of } \mathbb{R}^2.$$

(b) Find a basis for the range of T.

(c) Find a basis for the kernel of T. Make sure to write each element of your basis as a polynomial.

12. (14 points) Consider the system of equations

$$\begin{cases} sx_1 & - & 3x_2 & = 2 \\ 3x_1 & - & sx_2 & = 3 \end{cases}$$

where s is a number.

(a) For values of s for which the system has a unique solution, USE CRAMER'S RULE to find the solution, giving explicit formulas for x_1 and x_2 in terms of s.

(b) For which values of s does the system have a unique solution?