**Math 20580** Name:____________________________
**Midterm 1** Instructor:____________________________
**September 21, 2021** Section:____________________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

**Sign the pledge.** "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e

2. a b c d e

3. a b c d e

4. a b c d e

5. a b c d e

6. a b c d e

7. a b c d e

8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

## Part I: Multiple choice questions (7 points each)

1. If the matrices $A, B$ are such that

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \qquad BA = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix},$$

then what is the matrix $B$?

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 5 \\ -11 & 8 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (e) It can't be determined.

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 5 \end{bmatrix}.$$

Which of the following vectors are in the null space of $A$?

(I) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (II) $\begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$ (III) $\begin{bmatrix} 5 \\ -5 \\ 1 \end{bmatrix}$ (IV) $\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

(a) (IV) only

(b) (I) and (III) only

(c) (I) and (II) only

(d) (I) and (IV) only

(e) (II), (III) and (IV) only

3. Consider the following vectors.

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \vec{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \vec{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$

Which of the following sets of vectors is linearly independent?

(a) $\{\vec{u}, \vec{w}\}$

(b) $\{\vec{u}, \vec{v}, \vec{x}\}$

(c) $\{\vec{v}, \vec{x}\}$

(d) $\{\vec{u}, \vec{v}\}$

(e) $\{\vec{u}, \vec{v}, \vec{w}, \vec{x}\}$

4. Suppose that the null space of a matrix $A$ has dimension 3. Which of the following *must* be true about $A$?

(a) $A\vec{x} = \vec{b}$ always has a solution for any $\vec{b}$.

(b) $A\vec{x} = \vec{b}$ never has a solution for any $\vec{b}$.

(c) If $A\vec{x} = \vec{b}$ has a solution, it will be unique.

(d) If $A\vec{x} = \vec{b}$ has a solution, it will not be unique.

(e) For some $\vec{b}$, $A\vec{x} = \vec{b}$ has exactly 3 solutions.

5. What can be said about the following system of linear equations?

$$\begin{cases} 4x_1 + 6x_3 = 0 \\ 11x_2 - 15x_3 = 0 \end{cases}$$

(a) The solution set is a subspace of $\mathbb{R}^3$ (b) The system is inconsistent

(c) There are only finitely many solutions (d) Every solution is in $\mathbb{R}^2$

(e) none of the above

6. Find the values of $k$ for which the columns of the matrix

$$A = \begin{bmatrix} 1 & 2 & k \\ 2 & k^2 & 4 \end{bmatrix}$$

do not span $\mathbb{R}^2$.

(a) $k = 2$ or $k = -2$ (b) $k = 0$ (c) $k = -1$ (d) $k = 2$

(e) none of the above

7. Consider a basis $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ of $\mathbb{R}^3$, and a linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ with the property that

$$T(\vec{b}_1) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \qquad T(\vec{b}_2) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \qquad T(\vec{b}_3) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

If $\vec{u}$ has coordinate vector $[\vec{u}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$ relative to $\mathcal{B}$, then $T(\vec{u})$ is equal to

(a) $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ (c) $2\vec{b}_1 + 4\vec{b}_2 + \vec{b}_3$ (d) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$

8. Let $A$ be a $4 \times 8$ matrix of rank 2. Which of the following is the dimension of the null space of $A$?

(a) 0 (b) 2 (c) 4 (d) 6 (e) 8

**Part II: Partial credit questions (11 points each). Show your work.**

9. Consider the linear system

$$\begin{cases} x_1 - x_2 + 3x_4 + x_5 = 2 \\ x_1 + x_2 + 2x_3 + x_4 - x_5 = 4 \\ x_2 + 2x_4 + 3x_5 = 0 \end{cases}$$

(a) Write down the coefficient matrix and the augmented matrix of the system.

(b) Describe the solution set of the system in parametric vector form.

10. Find the inverse of the matrix

$$\begin{bmatrix} 0 & 1 & -2 \\ 1 & 5 & -4 \\ 0 & 1 & -1 \end{bmatrix}$$

11. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{bmatrix}$$

(a) Find the rank of $A$, find a basis $\mathcal{B}$ for $\mathrm{Col}(A)$, and find a basis $\mathcal{R}$ for $\mathrm{Row}(A)$.

(b) For the basis $\mathcal{B}$ found in part (a), determine the coordinate vector $[\vec{v}]_{\mathcal{B}}$, where $\vec{v} = \vec{a}_2 + 2\vec{a}_3 - \vec{a}_4$ and $\vec{a}_i$ denotes the $i$-th column of the matrix $A$.

12. (a) Find a $3 \times 3$ matrix whose null space contains all the vectors of the form $\begin{bmatrix} r \\ r \\ r \end{bmatrix}$.

(b) Find a $3 \times 3$ matrix whose column space contains all the vectors of the form $\begin{bmatrix} s+t \\ s-t \\ 2t \end{bmatrix}$.

(c) Find a $3 \times 3$ matrix whose null space consists of all vectors of the form $\begin{bmatrix} r \\ r \\ r \end{bmatrix}$, and whose column space consists of all vectors of the form $\begin{bmatrix} s+t \\ s-t \\ 2t \end{bmatrix}$.