Name: Date: 01/23/2025

Math 20580 (L.A. and D.E.) Tutorial
Quiz 1

1. Given the augmented matrix below corresponding to a linear system of equations, determine how many solutions the system has (if any). Also, if the corresponding linear system is consistent, determine the number of free variables it has.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 1 & 3 & 4 & 1 \\ 0 & 1 & 3 & -2 \end{array}\right]$$

$R_2 - R_1$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 3 & -2 \\ 0 & 1 & 3 & -2 \end{array}\right]$ $R_3 - R_2$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Pivots. Free variable since no pivot.

Consistent since no row of $\left[\begin{array}{ccc|c} 0 & 0 & 0 & x \end{array}\right]$, $x \neq 0$

There's a free variable so there is ∞-many solutions.

2. Determine the general solution to the system of equations below.

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ 6x_1 + 4x_2 &= 1. \end{aligned}$$

This linear system of equations corresponds to the augmented matrix

$\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 6 & 4 & 1 \end{array}\right]$ $R_2 - 3R_1$ $\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & 1 & -8 \end{array}\right]$ $R_1 - R_2$ $\left[\begin{array}{cc|c} 2 & 0 & 12 \\ 0 & 1 & -8 \end{array}\right]$

So $x_2 = -8$ and $2x_1 = 12 \Rightarrow x_1 = 6$

$x_1 = 6$

$x_2 = -8$