Name: Date: 01/20/2022

Math 20580 (L.A. and D.E.) Tutorial
Quiz Week 2

1. Determine whether or not

$$\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right).$$

Solution: Set up the augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 5 \\ 0 & 0 & 1 & -2 \\ 1 & 2 & 1 & 1 \end{array}\right]$$

Do EROs to get the row echelon form:

$$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 5 \\ 0 & 0 & 1 & -2 \\ 1 & 2 & 1 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & -2 \\ 2 & 3 & 0 & 5 \end{array}\right] \xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & -1 & -2 & 3 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & -2 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Since the rank of the coefficient matrix equals to the rank of the augmented matrix, the system is consistent. The answer is YES.

2. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix}$. If possible, compute $A^T A + 6B^{-1}$.

Solution:

$$\begin{aligned} A^T A + 6B^{-1} &= \begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \end{bmatrix} + \frac{6}{2} \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 14 \\ 14 & 21 \end{bmatrix} + \begin{bmatrix} 6 & 12 \\ 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 16 & 26 \\ 14 & 24 \end{bmatrix} \end{aligned}$$