Name: Date: 02/06/2025

Math 20580 (L.A. and D.E.) Tutorial
Quiz Week 3

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be

$$T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x - 2y \\ 5x - 7y \\ y \end{bmatrix}$$

Determine whether or not T is linear.

Solution

First, for any $c \in \mathbb{R}$,

$$T(c\begin{bmatrix} x \\ y \end{bmatrix}) = T(\begin{bmatrix} cx \\ cy \end{bmatrix}) = \begin{bmatrix} cx - 2cy \\ 5cx - 7cy \\ cy \end{bmatrix} = c\begin{bmatrix} x - 2y \\ 5x - 7y \\ y \end{bmatrix} = cT(\begin{bmatrix} x \\ y \end{bmatrix}).$$

Second, suppose $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$. Then

$$\begin{aligned} T(\vec{v} + \vec{w}) = T(\begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix}) &= \begin{bmatrix} (v_1 + w_1) - 2(v_2 + w_2) \\ 5(v_1 + w_1) - 7(v_2 + w_2) \\ v_2 + w_2 \end{bmatrix} \\ &= \begin{bmatrix} v_1 - 2v_2 \\ 5v_1 - 7v_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} w_1 - 2w_2 \\ 5w_1 - 7w_2 \\ w_2 \end{bmatrix} = T(\vec{v}) + T(\vec{w}). \end{aligned}$$

Hence, $T$is linear.

2. Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 1 & -4 & 4 \end{bmatrix}$ and $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$ be following.

$$\vec{v_1} = \begin{bmatrix} -4 \\ 5 \\ 6 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \vec{v_4} = \begin{bmatrix} 7 \\ 0 \\ 4 \end{bmatrix}.$$

Determine which of the vectors are among $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$ in Null(A).

Solution

Recall that the null space of A is the subspace of $\mathbb{R}$ consisting of solutions of the homogeneous linear system $Ax = 0$. It is denoted by null (A). In order for a vector $\vec{v}$ to belong to null (A), we must check that $A \cdot v = 0$. After doing the computations, we see that $A\vec{v_1} = A\vec{v_3} = \vec{0}$. Moreover, $A\vec{v_2} \neq \vec{0}$ and $A\vec{v_4} \neq \vec{0}$. Therefore, the only vectors that belong to null (A) are $\vec{v_1}$ and $\vec{v_3}$.