Name: Date: 02/20/2025

Math 20580 (L.A. and D.E.) Tutorial
Quiz 4

1. Is the set of 2×2 diagonal matrices with real entries

$$\mathcal{D}_2 = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

a subspace of $\mathcal{M}_{2\times 2}(\mathbb{R})$?

Solution: Yes.

- Observe that the zero matrix is a diagonal matrix with $a = b = 0$. Then $0 \in \mathcal{D}_2$.
- Let $A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}$ in $\mathcal{D}_2$. We have

$$A + B = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & 0 \\ 0 & a_2 + b_2 \end{bmatrix} \in \mathcal{D}_2$$

- Let $c \in \mathbb{R}$ and $A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \in \mathcal{D}_2$. Then,

$$cA = c \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} = \begin{bmatrix} ca_1 & 0 \\ 0 & ca_2 \end{bmatrix} \in \mathcal{D}_2$$

Hence, $\mathcal{D}_2$ is a subspace of $\mathbb{M}_2(\mathbb{R})$.

2. Determine whether the set $\{1 - x, 1 - x^2, x - x^2\}$ is a basis for $\mathcal{P}_2$.

Solution: No, they are linearly dependent because

$$(1 - x) + (x - x^2) = 1 - x^2.$$