Name: Date: 02/27/2025

M20580 L.A. and D.E.
Quiz 5

1. Let $T : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}$ defined as,

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = b - 2c$$

Write a basis for $\ker(T)$.

Solution: If $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = 0$, then $b - 2a = 0$ or $a = 2b$.
Hence

$$\ker(T) = \left\{ \begin{bmatrix} a & 2c \\ c & d \end{bmatrix} a, c, d \in \mathbb{R} \right\}$$

Note,

$$\begin{bmatrix} a & 2c \\ c & d \end{bmatrix} = a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} + d \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Basis for $\ker(T) = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

2. Let $F : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$

$$F(p(x)) = p'(x) + 2p(x).$$

Write the matrix corresponding to the linear transformation F with respect to the standard basis $\mathcal{E} = \{1, x, x^2\}$.

Solution: To compute the matrix corresponding to the linear Transformation, note, $F(1) = 2,\ F(x) = 1 + 2x,\ F(x^2) = 2x + 2x^2$. Hence

$$[F]_{\mathcal{E}\leftarrow\mathcal{E}} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}.$$