M20580 L.A. and D.E.
Quiz 6

1. Find the determinant of the following 4×4 matrix by expanding along the third row:

$$\begin{bmatrix} 0 & 2 & -1 & 1 \\ 1 & 3 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 2 & -1 & 1 & 0 \end{bmatrix}$$

Solution: We use cofactor expansion along the third row. Since the only non-zero term is 2,

$$\det \begin{bmatrix} 0 & 2 & -1 & 1 \\ 1 & 3 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 2 & -1 & 1 & 0 \end{bmatrix} = (-1)^{3+2} \cdot 2\det \begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}.$$

Then using cofactor expansion again along the first row, we have

$$= -2 \left[(-1)^{1+2}(-1)\det \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + (-1)^{1+3} \cdot 1\det \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \right].$$

The final answer is then 6.

2. Consider the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$. Is $\mathbf{x} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ an eigenvector of A? If so, what is its associated eigenvalue?

Solution: If x is an eigenvector then that means we can find some λ such that $Ax = \lambda x$. We compute

$$Ax = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 16 \end{bmatrix} = 4\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

So, $x = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ is an eigenvector of A and its associated eigenvalue is 4.