

M20580 L.A. and D.E. Tutorial
Quiz 7

1. Let A be a 3×3 matrix. Suppose that A is diagonalizable and the eigenvalues of A are $2, -1, 0$ with eigenvectors

$$v_{-1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

respectively. Find A^3 .

Hint: Use $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and $P^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$.

Solution: Since A is diagonalizable, then

$$A = P \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} P^{-1}$$

and

$$A^3 = P \begin{bmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} P^{-1} = \frac{1}{2} \begin{bmatrix} 7 & -9 & 9 \\ -1 & -1 & 1 \\ 8 & -8 & 8 \end{bmatrix}.$$

2. Let $W = \text{span} \left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \right\}$. Which of the following vectors is in W^\perp ?

(a) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$ (c) $\begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

Solution: Since the vector is in W^\perp if and only if it is orthogonal to both $\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, only (d) is possible.