Name: SOLUTIONS Date:

M20580 L.A. and D.E.
Quiz 9

1. Find the least square solutions to $A\mathbf{x} = \mathbf{b}$ where,

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 0 \\ 1 & 3 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 5 \\ -1 \\ 3 \\ 0 \end{bmatrix}.$$

> ***Solution:*** This is equivalent to solving the system,
>
> $$A^\top A\mathbf{x} = A^\top \mathbf{b}$$
>
> Computing,
>
> $$A^\top A = \begin{bmatrix} 6 & 6 \\ 6 & 11 \end{bmatrix}, \ A^\top \mathbf{b} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$
>
> Hence,
>
> $$\begin{aligned} \mathbf{x} &= (A^\top A)^{-1} A^\top \mathbf{b} \\ &= \begin{bmatrix} 7 & 6 \\ 6 & 11 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 11 & -6 \\ -6 & 7 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix} \\ &= \frac{1}{41} \begin{bmatrix} 42 \\ -8 \end{bmatrix} \end{aligned}$$

2. Solve the following initial value problem,

$$\frac{dy}{dx} = \sin(2x), \ y(0) = 0.$$

Is the solution unique?

> ***Solution:*** Seperating variable and integrating on both sides we get
>
> $$y(x) = \frac{-\cos(2x)}{2} + c$$
>
> substituting $y(0) = 0$ we get,
>
> $$0 = \frac{-1}{2} + c$$
>
> and hence $c = \frac{1}{2}$.
>
> $$y(x) = \frac{1 - \cos(2x)}{2}.$$
>
> The solution is well defined for all $\mathbb{R}$ and hence the IVP is unique.