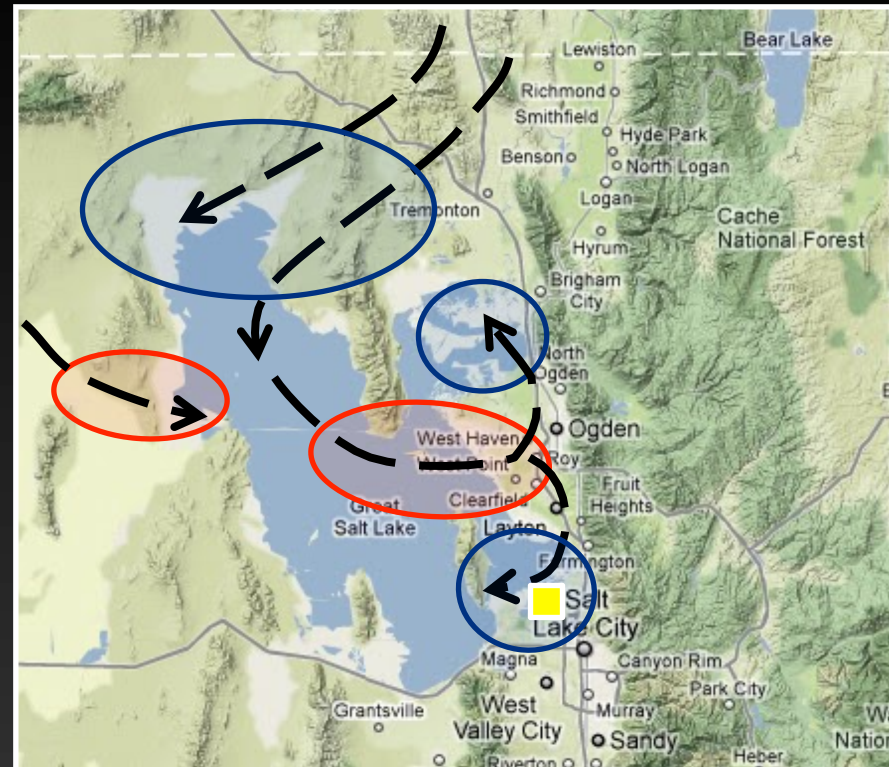
The Potential Utility of High Resolution Ensemble Sensitivities During Weak Flow in Complex Terrain

Joshua Hacker



A winter-time advection fog event at the Salt Lake airport

Ensemble sensitivity

How does the change in a set of initial state variables $\mathbf{x}_s$ change a forecast metric J?

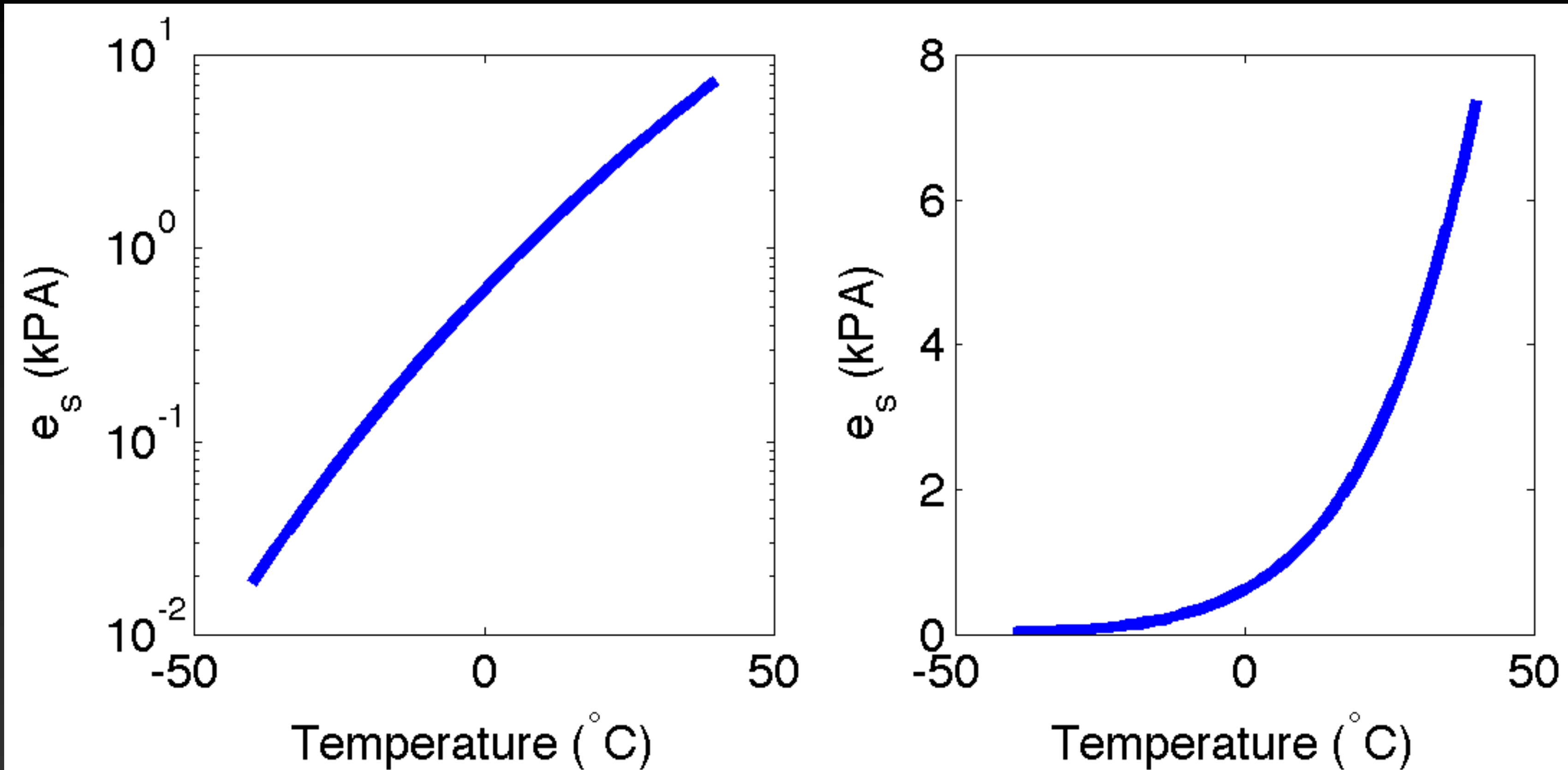
$$\frac{\partial J_e}{\partial x^a}$$

- Identify dynamically relevant covariance structures in space and time, and over complex terrain
- Propose observing strategies for mesoscale, short-range forecasts in complex terrain
- Sensitivity scales (time and space) to infer predictability scales
- Predictability of specific phenomena

Open issues:

- Linearity assumptions in complex terrain
- Linearity assumptions at fine scales
- Sampling error
- Effects of ignoring cross-variable covariances

Physical linearity



Teten's formula (both panels) nonlinear, but is approximately linear across small temperature ranges.

Statistical/Dynamical Linearity

$$\mathbf{J}_e = \left[\mathbf{X}^a\right]^{\mathrm{T}} \boldsymbol{\beta} + \varepsilon$$

$$\hat{\boldsymbol{\beta}} = \frac{\partial J_e}{\partial \mathbf{x}^a} = \mathbf{X}^a \left(\left[\mathbf{X}^a\right]^{\mathrm{T}} \mathbf{X}^a\right)^{-1} \mathbf{J}_e = \mathbf{Q}\mathbf{R}^{-\mathrm{T}} \mathbf{J}_e$$

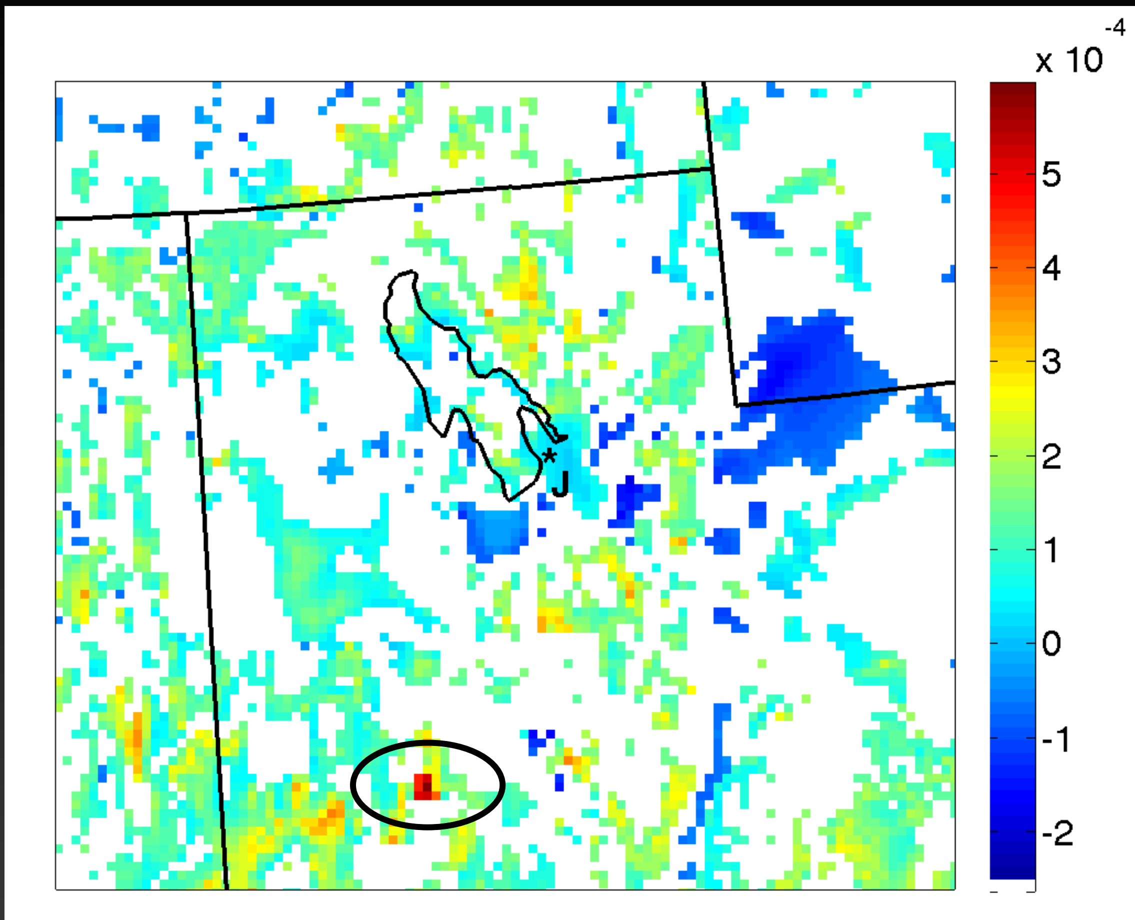
$\mathbf{J}_e$ are statistical perturbations about J_e

$\mathbf{X}^a$ are statistical perturbations about $\mathbf{x}^a$

Formal sensitivity is multi-variate linear regression; coefficients can be estimated via a right pseudo-inverse.

- Assumes Gaussian distributions and linear relationship.
- In meteorological literature, covariance is always approximated by diagonal (makes inversion trivial).
- In either case, linearity formally valid only for small perturbations about $\mathbf{x}^a$.

Sensitivity of Q_v to θ



$$\frac{\partial J_e}{\partial \mathbf{x}^{\mathbf{a}}} \text{ units}\left[\frac{kg}{kg\ K}\right]$$

- Warm colors show predictions that a positive θ perturbation there will increase water vapor mixing ratio over KSLC.
- Stronger inversion shown by warmer temperatures at higher elevations.

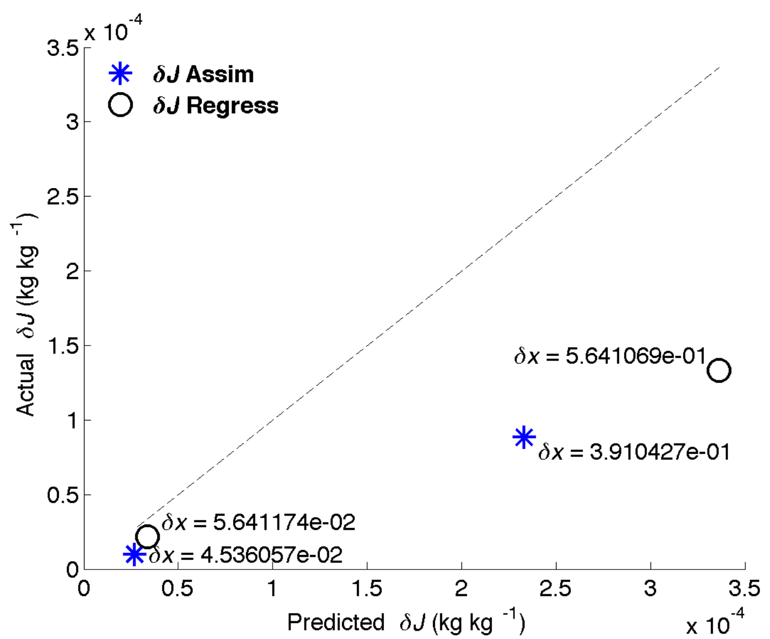
Prediction Tests

$$\delta J_e = \frac{\partial J_e}{\partial \mathbf{x}^a}\delta \mathbf{x}^a \text{ predicted forecast change}$$

$$\Delta J_e = \mathrm{J}\left[\mathrm{M}(\mathbf{x}^a + \delta \mathbf{x}^a)\right] \text{ change from nonlinear integration}$$

J is function of nonlinear model forecast M.

- Comparison of δJ *vs.* ΔJ indicates accuracy of linear approximations from sample statistics
- Control analysis at sensitivity point: $\sigma_\theta = 0.0516$ K
- Over-prediction can result from sampling error



Approximation

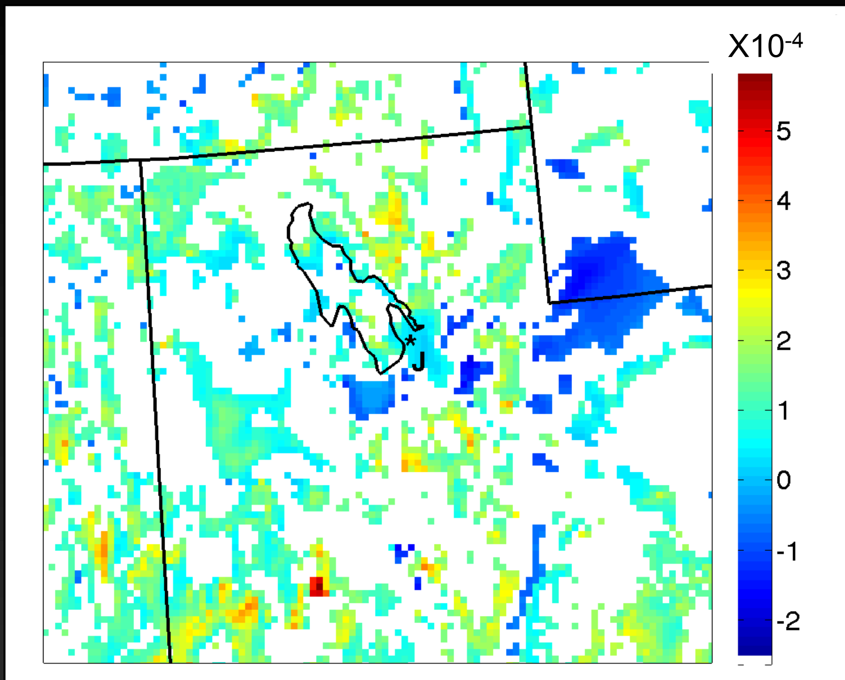
$$\frac{\partial J_e}{\partial \mathbf{x}^a} = \left[\mathbf{P}^a\right]^{-1} \mathbf{X}^a \mathbf{J}_e \approx \left[\mathbf{D}^a\right]^{-1} \mathbf{X}^a \mathbf{J}_e$$

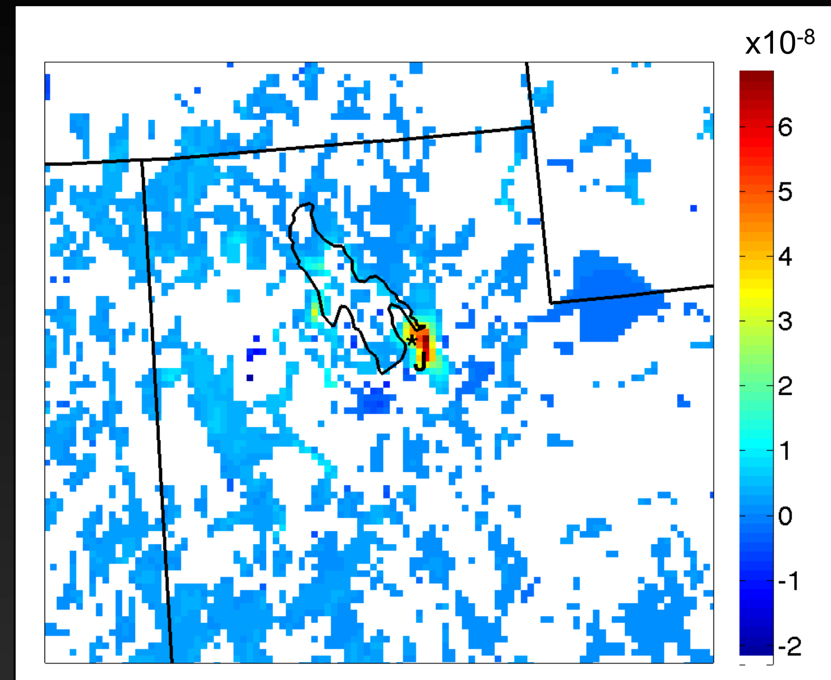
$$\mathbf{P}^a = \mathbf{X}^a \left[\mathbf{X}^a\right]^{\mathrm{T}}, \ \mathbf{D}^a = \mathrm{diag}\left(\mathbf{P}^a\right)$$

Approximate sensitivity more common in the literature avoids an inversion by assuming covariances are zero, leading to a scalar problem for each state element.

Better approximation from diagonal expected for smoother fields with spatially coherent regions of strong correlation $(\mathbf{x}^a, \mathbf{J})$.

Approx. vs. full covariance





$$\frac{\partial J_e}{\partial \mathbf{x}^{\mathbf{a}}} \text{ units} \left[\frac{kg}{kg\ K}\right]$$

Structures are broadly similar, but greatest sensitivity located near J. Sensitivities orders of magnitude smaller because all grid points can contribute instead of assuming one.

Summary

- Several open issues remain with using ensemble sensitivities at mesoscales and in complex terrain: linearity, sampling error, approximations
- To first order, results show that they can be used effectively.
- Care needed for handling regressions; perhaps consider localization to handle sampling error.

Acknowledgements

- Office of Naval Research MURI program (MATERHORN)
- Data Assimilation Research Testbed (DART) team
- Weather Research and Forecasting (WRF) team