Improving the Immersed Boundary Method in WRF for Complex Mountainous Terrain

Implementation of surface scalar and momentum fluxes for WRF-IBM

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Limitations of WRF

• WRF: Weather Research and Forecasting model
• Terrain following coordinate system
  – Horizontal gradient errors
  – Numerical instability
  – Terrain slope limit
  – Grid aspect ratio limit
Higher resolution – Steeper slopes

\[ dx = 500\text{m} \]
\[ \text{max slope} \sim 20\text{ degrees} \]
Higher resolution – Steeper slopes

dx = 60m
max slope ~ 70 degrees
Immersed boundary method

Terrain following coordinates

Immersed boundary (WRF-IBM)
WRF implementation of scalar and momentum flux

- Advection diffusion equation for scalar
  \[
  \frac{\partial T}{\partial t} + V \cdot \nabla T = \left( \frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} + \frac{\partial H_3}{\partial z} \right) + F_T
  \]

- Momentum equation in U direction
  \[
  \frac{U}{t} + U \frac{U}{x} + V \frac{U}{y} + W \frac{U}{z} = \frac{1}{x} \frac{P}{x} \left( \frac{11}{x} + \frac{12}{y} + \frac{13}{z} \right)
  \]

- Requires gradient in \( H_3 \) and \( H_3 \)
WRF implementation of scalar flux

\[
\frac{\partial T_1}{\partial t} = \ldots \quad \frac{\partial H_3}{\partial z} = \ldots \quad \frac{H_3\big|_2}{z} \quad \frac{H_{\text{surface}}}{z}
\]

\[
H_3\big|_2 = k_T \frac{\partial T}{\partial z} = k_T \frac{T_2 - T_1}{z}
\]
WRF implementation of momentum flux

Log Law

\[
\frac{\partial U}{\partial t} = \frac{\partial_1}{\partial z} = \frac{13_2}{z} \quad \text{wall}
\]

\[
13_2 = \nu_T \frac{\partial U}{\partial z} = \nu_T \frac{U_2}{z} \frac{U_1}{z}
\]

\[
\text{wall} = u_*^2 = C_d \left| U_1 \right| U_1
\]
Difficulty of WRF-IBM scalar flux implementation

- Potential temperature is updated as: \( \frac{\partial T_1}{\partial t} = \ldots \frac{H_{3|_2}}{H_{3|_1}} \)
- Correct \( H_{3|_1} \) is required
- WRF: \( H_{3|_1} = H_{\text{surface}} \)  WRF-IBM: \( H_{3|_1} = k_{T(\alpha H_{3|_1})} \frac{(T_1 - T_g)}{z} \)
- Correct \( T_g \) and \( k_{T(\alpha H_{3|_1})} \) is required for WRF-IBM
IBM implementation of heat flux

\[ \frac{\partial T_1}{\partial t} = \frac{\partial H_3}{\partial z} = \frac{H_3|_2}{z} - \frac{H_3|_1}{z} \ldots \]

Need correct \(H_3|_1\)

\[ H_3|_1 = k_T(atH_3|_1) \frac{T_1}{T_g} \]

Need correct \(T_g\) and \(k_T(atH_3|_1)\)

\[ hfx = k_T(wall) \frac{T_i}{T_g} \]

\[ T_g = T_i + \frac{hfx}{k_T(wall)} \frac{z}{z} \]

Need correct \(k_T(wall)\) and \(k_T(atH_3|_1)\)
IBM implementation of heat flux

Need correct $k_{T(wall)}$

Method: Prandtl’s mixing length
- More realistic simulation
- Can couple with turbulence closure

$v_{T(wall)} = u_*kz$

$k_{T(wall)} = \frac{v_{T(wall)}}{Pr}$
IBM implementation of heat flux

Need correct \( k_{T(atH_3)} \)

Method: Prandtl’s mixing length
- More realistic simulation
- Can couple with turbulence closure

\[
v_{T(atH_3)} = u_k z(atH_3) \]

\[
k_{T(atH_3)} = \frac{v_{T(atH_3)}}{\text{Pr}} \]
Idealized thermal driven flow simulation

- Uncoupled simulations with specified surface heating
- Coupled simulations using atmospheric parameterizations
### Idealized validation cases summary

<table>
<thead>
<tr>
<th></th>
<th>Flat plate(a) /Idealized valley(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1</strong></td>
<td>Uncoupled</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td>Coupled</td>
</tr>
</tbody>
</table>

Prandtl’s mixing length:

\[
T_{(wall)} = \frac{u_* k z}{Pr} \quad T_{(H_3|1)} = \frac{u_* k z (atH_3|1)}{Pr}
\]

- Can couple with different turbulence closure
- Matches perfectly for flat plate
- Idealized valley simulation is still under work
Cases 1a and 2a - Flat plate setup

Domain Set-Up
• \((X,Y,Z) = (1, 1, 6)\) km
• \(\Delta X = \Delta Y = 100 \text{ m}, \Delta Z \sim 100 \text{ m}\)

Initialization
• \((U,V,W) = (1,0,0)\)
• Neutral mixed layer and capping inversion at top

Simulation
• 6:00 to 18:00 UTC
• Uncoupled and Coupled (RRTM Longwave Radiation/MM5 Shortwave Radiation/MM5 Surface Layer Model/NOAH Land Surface Model)
  • Smagorinsky closure
  • Free slip bottom boundary condition
Uncoupled Flat plate potential temperature

- **WRF-IBM (blue)** and **WRF (red)**
Coupled flat plate potential temperature

- **WRF-IBM (blue)** and **WRF (red)**
Coupled flat plate
radiation/surface physics

- **WRF-IBM (blue)** and **WRF (red)**
- Excellent agreement
Neutral boundary layer setup

- Geostrophically forced flow over a flat plate
- $U_g = 10\text{m/s}$, $V_g = 0\text{m/s}$
- $dx = dy = 32\text{m}$
- Domain size $\sim 1500\text{m}$ in each direction
- Plate located at $100\text{m}$
- Log law at bottom boundary
- Smagorinsky turbulence closure
- Turbulence introduced at initialization

- 2 WRF-IBM cases and 1 WRF case with different vertical levels
Grid setup

<table>
<thead>
<tr>
<th></th>
<th>Zlevel</th>
<th>Z_top</th>
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<tbody>
<tr>
<td>WRF</td>
<td>42</td>
<td>1500</td>
</tr>
<tr>
<td>IBM_1</td>
<td>45</td>
<td>1500</td>
</tr>
<tr>
<td>IBM_2</td>
<td>50</td>
<td>1500</td>
</tr>
</tbody>
</table>
WRF and WRF-IBM velocity profiles

- IBM-WRF and WRF match
Comparison to log law profile
Nondimensional shear profile

- More sensitive measure of log law performance
- Nondimensional velocity gradient

\[ F = \frac{z}{U_*} \sqrt{\left( \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2} \]

- In the logarithmic region of NABL
  \[ = 1 \]
Ongoing work

• Ongoing test cases including idealized valley for both surface scalar and momentum fluxes
• More test cases for stable/unstable cases
• Implementation with higher order turbulence closures including TKE 1.5
Conclusions

• Verification of the implementation of surface heat flux boundary condition in WRF-IBM
• Verification of the immersed boundary method with log law wall model for neutral boundary layer for momentum
• WRF-IBM agrees well with WRF results.
• Different turbulence models can be coupled with the bottom boundary condition
Difficulty of WRF-IBM momentum implementation

- Potential temperature is updated as: \( \frac{\partial U}{\partial t} = \ldots \)
- Correct \( ^{13}_1 \) is required
- WRF: \( ^{13}_1 = \text{wall} = C_d |U_1| U_1 \)
- WRF-IBM: \( ^{13}_1 = v_{T(at \ 3\|)} \frac{(U_1 U_g)}{z} \)
- Correct \( U_g \) and \( v_{T(at \ 3\|)} \) is required for WRF-IBM
IBM implementation of momentum flux

Log Law

\[ \begin{align*}
\frac{\partial U_1}{\partial t} &= \ldots \quad \frac{\partial}{\partial z} U_{13} = \ldots \\
\frac{13}{2} &= \frac{13}{1} \\
\end{align*} \]

Need correct \[ \frac{13}{1} \]

\[ \begin{align*}
13 |_1 = v_{t(at)} \frac{U_1}{U_g} \frac{U_i}{z} \\
\end{align*} \]

Need correct \( U_g \) and \( v_{t(at)} \)

\[ \begin{align*}
wall = v_{t(wall)} \frac{U_i}{U_g} \frac{U_i}{z} \\
\end{align*} \]

\[ \begin{align*}
U_g &= U_i + \frac{wall}{z} \\
\end{align*} \]

Need correct \( v_{t(wall)} \) and \( v_{t(at)} \)
IBM implementation of momentum flux

Log Law

Need correct $v_{t(wall)}$

Method: Prandtl's mixing length
- More realistic simulation
- Can couple with turbulence closure

$v_{t(wall)} = u_* k z$
IBM implementation of momentum flux

Log law

Need correct $v_t(atH_{3|1})$

Method: Prandtl’s mixing length
- More realistic simulation
- Can couple with turbulence closure

$v_T(atH_{3|1}) = u_\star k z(at_{13|1})$