On the Welfare and Cyclical Implications of Moderate Trend Inflation*

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Abstract

We address the welfare and cyclical implications of moderate trend inflation in an augmented medium-scale DSGE model. In this framework, increasing trend inflation from 2 to 4 percent, in accordance with some recent proposals, would generate a consumption-equivalent welfare loss of 3.7 percent based on the non-stochastic steady state and 4.3 percent based on the stochastic mean. Welfare costs of such a high magnitude are driven by five main factors: i) staggered wage contracts, ii) trend growth in investment-specific and neutral technology, iii) extended borrowing, iv) roundaboutness in U.S. production, and v) and the interaction between trend inflation and shocks to the marginal efficiency of investment (MEI). In contrast, a sticky-price model abstracting from these other features would generate corresponding welfare losses of only 0.17 percent and 0.22 percent, respectively. In our framework, moderate trend inflation has important business-cycle implications, interacting much more strongly with MEI shocks than with either productivity or monetary shocks. Our model also avoids the short-run decline in consumption following a positive MEI shock which is typically found in New Keynesian models and thus accounts for a simultaneous increase in consumption, hours and output mainly due to the presence of economic growth and roundabout production.

JEL classification: E31, E32.

Keywords: Wage and price contracting; trend inflation; trend growth in technology; financial intermediation; roundabout production; investment shocks; inflation costs; business cycles.

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1 Introduction

In the aftermath of the Great Recession, a number of economists have argued that the Federal Reserve and other central banks should raise their inflation targets. At the time of the recession, the consensus was that the inflation target was about 2 percent annually. Economists like Blanchard, Dell’Ariccia, and Mauro (2010), Ball (2013), and Krugman (2014) have since advocated for increases in the inflation target to 4 or even 5 percent. Implementing such proposals over a sufficiently long period of time would eventually lead to higher long-run or trend inflation. Proposals to raise the inflation target therefore naturally lead to the following question: how would the U.S. economy be affected by a moderate rise in trend inflation? Our paper offers a new perspective on this policy question. While doing so, it also provides new insights on the effects of moderate trend inflation on the welfare and business-cycle properties of medium-scale New Keynesian models more generally.

There exists a large literature that studies the macroeconomic consequences of non-zero trend inflation. So far, the majority of models used for that purpose have been small-scale sticky-price models with positive trend inflation (e.g. see Ascari, 2004; Hornstein and Wolman, 2005; Kiley, 2007; Levin and Yun, 2007; Amano, Ambler, and Rebei, 2007; Ascaari and Ropele, 2007; Coibion and Gorodnichenko, 2011).1 By “small-scale” we mean that these models abstract from capital accumulation and most forms of real rigidity. By “sticky-price” we mean that these models typically assume flexible nominal wages. Two partial exceptions are Amano, Ambler, and Rebei (2007) and Amano, Moran, Murchison, and Rennison (2009). The former features a model with capital and convex capital adjustment costs, but abstracts from wage rigidity. The latter considers price and wage rigidity together, but omits capital and real rigidities.

In this existing literature, a trend inflation rate of less than 4 percent generally has a modest impact on the properties of the standard New Keynesian model. For example, based on the canonical New Keynesian model with sticky prices only, Ascari (2004) finds that a rise in trend inflation from 2 to 4 percent generates an additional steady-state output loss of about 0.5 percent. Amano et al. (2009) find that raising trend inflation from 2 to 4 percent generates an additional consumption-equivalent welfare loss of less than one percent. Using alternative versions of a sticky-price model, Amano, Ambler and Rebei (2007), Ascari and Ropele (2007) and Ascari and Sbordone (2014) show that an inflation trend of less than 4 percent has a modest impact on business-cycle fluctuations.2

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1 For a survey of this literature, see Ascari and Sbordone (2014) and the references therein.
2 Other questions addressed in this literature are the effect of trend inflation on the Taylor principle and the determinacy region more generally (Hornstein and Wolman, 2005; Kiley, 2007; Ascaari and Ropele, 2009; Coibion and Gorodnichenko, 2011), and the link between trend inflation and optimal monetary policy (Ascaari and Ropele, 2007).
We contribute to this literature by focusing on an augmented medium-scale New Keynesian model of the type first popularized by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). Our paper is a first attempt to look at how moderate trend inflation affects the welfare and business cycle properties of these medium-scale models. The model we develop in this paper rests on several key features, some of which are relatively common ingredients in medium-scale models and others which are not. First, our model combines non-zero steady-state inflation with nominal wage and price rigidities in the form of Calvo staggered contracts. Second, it includes real frictions – namely habit formation in preferences for consumption, investment adjustment costs, and variable capital utilization. Third, it embeds real per capita output growth stemming from trend growth in neutral and investment-specific technologies. Fourth, it incorporates a roundabout production structure. Fifth, it features an extended working capital channel, wherein firms borrow funds from a financial intermediary to finance the costs of all of their variable inputs and not only their wage bill. Sixth, the monetary authority sets nominal interest rates based on an inertial Taylor-type of rule. Finally, along with productivity and monetary policy shocks, our model includes an important shock to the marginal efficiency of investment (henceforth MEI) as in Justiniano, Primiceri and Tambalotti (2011) which, unlike other types of shocks, interacts quite strongly with trend inflation. Our paper is the first to shed light on this interaction between trend inflation and MEI shocks.

The first part of the paper provides a quantitative evaluation of the welfare costs of positive trend inflation. The second part examines the effects of moderate trend inflation on the business cycle. When conducting our welfare analysis, we focus on a change in trend inflation from 2 to 4 percent, a scenario consistent with recent proposals to raise the inflation target. Also since the average rate of inflation has declined from about 4 percent (annualized) during the years 1960-1983 to 2 percent on average since 1984, our evidence is suggestive of the welfare gains associated with the 2 percent decline in trend inflation. Our quantitative evaluation of the welfare costs of trend inflation rests on two measures: a consumption-equivalent welfare loss metric based on non-stochastic steady states and another metric calculated from stochastic means. When exploring the effects of moderate trend inflation on business-cycle fluctuations, we compare the impulse responses of key macroeconomic variables to alternative disturbances for trend inflation rates between 0 and 4 percent. We show that moderate trend inflation has surprisingly strong normative and positive implications.

This works builds on a revival of the idea that investment shocks stand as a plausible alternative to productivity shocks as the principal driver behind economic fluctuations. See also Greenwood, Hercowitz and Krussel (1997); Fisher (2006); Justiniano and Primiceri (2008); and Justiniano, Primiceri and Tambalotti (2011).
Our first set of substantive findings pertains to the normative implications of moderate trend inflation. Seen through our baseline model, the cost of raising trend inflation from 2 to 4 percent is 3.73 percent of each period’s consumption based on non-stochastic steady states and 4.3 percent based on stochastic means. Since these welfare losses are substantially higher than what much of the existing literature has found, it is only natural to ask what are the features of our model which are responsible for welfare costs of this magnitude. Our analysis points to five important factors: i) staggered wage contracts, ii) trend growth in investment-specific and neutral technology, iii) roundabout production, iv) extended working capital and v) the interaction between trend inflation and a persistent MEI shock. When our model is re-calibrated to abstract from these features, we find welfare costs of increasing the trend inflation rate from 2 to 4 percent of less than 0.25 percent of consumption, which is in line with the findings in much of the existing literature.

Staggered wage contracts are an important factor determining the welfare costs of trend inflation, significantly more than staggered price contracts. In particular, if we assumed that wages were flexible the welfare cost of raising trend inflation from 2 to 4 percent would only be 1.0 percent of consumption based on stochastic means - less than one-fourth of the consumption equivalent welfare loss as in our baseline analysis. Households with positive trend inflation would like to reset their wages each period, but only a fraction can. This leads to significant steady-state wage dispersion, which drives a wedge between aggregate labor supply and demand. It also results in higher wage markups on average, as updating households choose higher wages than they otherwise would to protect their future real wages from inflation. This higher average wage markup moves the economy further from the first best allocation, resulting in significant welfare losses. While qualitatively similar effects of trend inflation are also at work for price-setting, for plausible labor supply elasticities (e.g. non-infinite) the effect of trend inflation is substantially more important with wage-setting.

Trend growth in investment-specific and neutral technology, in conjunction with wage rigidity, also contributes significantly to the welfare costs of trend inflation. If there is no trend growth, then the welfare cost of moving from 2 to 4 percent inflation is significantly lower relative to our baseline – e.g. the cost of going from 2 to 4 percent trend inflation is about 2.5 percent of consumption based on stochastic means when there is no trend growth, compared to 4.3 percent when trend growth is positive. Positive trend growth means that households would like to adjust their wages each period even if trend inflation is zero. This results in steady state wage dispersion and higher than average wage markups than if trend growth were zero. Adding in positive trend inflation

\footnote{It turns out that the sources of trend growth – IST or neutral productivity – are equally important for this effect. Either source of growth implies that real wages grow along the balanced growth path, which means households would like to increase their nominal wages each period along that balanced growth path.}
exacerbates these distortions, resulting in much larger welfare costs than if there were no trend growth.

Roundabout production also plays a non negligible role in accounting for the welfare costs of positive trend inflation. The steady-state and mean consumption equivalent welfare costs of going from 2 to 4 percent trend inflation amount to 3.16 and 3.67 percent without roundabout production instead of 3.73 and 4.3 percent with it. Roundabout production has two effects in the model: it acts as an amplification source for real shocks and also is isomorphic to prices being stickier, because it introduces some strategic complementarity into price-setting (e.g. see Basu, 1995; Huang, Liu, and Phaneuf, 2004). Both of these features make trend inflation relatively more costly.

The extended working capital channel also contributes significantly to our findings on welfare costs. Without extended borrowing the steady-state and mean welfare costs decline to 3.27 and 3.69 percent, respectively. Intuitively, working capital must raise the costs of trend inflation, because higher trend inflation raises the average nominal interest rate, which effectively represents a direct distortion on the first-order conditions for optimal inputs.

Finally, there are also potentially interesting interactions between the various shock sources and the consumption equivalent welfare losses based on stochastic means. When our model is re-calibrated so as to exclude MEI shocks, the mean welfare cost of going from 2 to 4 percent trend inflation falls from 4.3 percent of consumption to 3.9 percent.

Our second set of findings pertains to the cyclical implications of positive trend inflation. Whereas trend inflation has relatively minor effects on the dynamic responses of aggregate variables to productivity and monetary shocks, there are large interactions between trend inflation and MEI shocks. Contrary to other types of shocks, the interaction between trend inflation and the cyclical responses to MEI shocks depends heavily on the persistence of the shock. For moderate levels of shock persistence, output reacts more strongly to a MEI shock the larger is trend inflation. In particular, under our baseline parameterization the impulse response of output at a ten quarter horizon is about 15 percent larger with 4 percent trend inflation compared to 2 percent trend inflation. Interestingly, the interaction between trend inflation and the MEI shock flips signs at higher levels of persistence. When the shock is sufficiently persistent, higher trend inflation significantly dampens the response of output and other aggregate variables to MEI shocks. For example, when the autoregressive parameter in the MEI process is 0.95 (instead of 0.81 in our baseline analysis), the output response is only one-third as large at a ten quarter horizon with 4 percent trend inflation compared to 2 percent trend inflation.

There is one final substantive finding emphasized in our paper. In the literature on investment-specific technology (IST) and marginal efficiency of investment (MEI) shocks (see Fisher, 2006
and Justiniano, Primiceri and Tambalotti, 2010 and 2011), an important concern is whether these shocks can generate a simultaneous increase in consumption, hours, and output. In standard neo-classical models consumption falls while investment and output rise after a positive MEI shock (or a positive investment-specific technology shock). In Justiniano, Primiceri and Tambalotti (2011), consumption initially declines after a positive MEI shock before taking about a year to rise. Interestingly, our model is able to avoid the short-run decline in consumption following an improvement to the marginal efficiency of investment which is typically found in New Keynesian models regardless of the level of trend inflation. That is, consumption increases immediately after a MEI shock. As a result, consumption is significantly more procyclical in our model than in other medium-scale scale DSGE models like those of Justiniano, Primiceri and Tambalotti (2010 and 2011). As we later show, the two important features for this finding relative to Justiniano, Primiceri and Tambalotti (2011) are trend growth and roundabout production.

The results of our paper have important implications for both policymakers and academics. On the policy front, the large welfare costs of trend inflation which we find represent a warning against policy proposals urging central banks to raise their inflation targets. In that respect, the message of our paper is complementary to Coibion, Gorodnichenko, and Wieland (2012), who weigh the benefits of a reduced incidence of zero lower bound episodes from higher trend inflation against the costs of higher trend inflation outside of periods where the zero lower bound binds. On the academic front, ours is the first paper to point out the large interaction between trend inflation and MEI shocks. An increasing body of research suggests that MEI shocks are a major driver of the business cycle – Justiniano, Primiceri, and Tambalotti (2010, 2011) find that these shocks account for 50 percent or more of the business cycle volatility in output. Conventional wisdom in the literature has been that trend inflation might matter in a normative sense, but that it is innocuous to ignore it for the purposes of understanding positive aspects of the business cycle. Our results suggest that this is not the case – trend inflation interacts strongly with MEI shocks, whether that effects is to dampen or amplify the effects of these shocks.

The remainder of the paper is organized as follows. Section 2 lays out our medium-scale DSGE model. Section 3 discusses some issues related to calibration. Section 4 examines the steady-state and mean welfare implications of moderate trend inflation. Section 5 studies the cyclical implications of trend inflation. Section 6 contains concluding remarks.
2 A Medium-Scale Macro Model with Trend Inflation

This section lays out our medium-scale DSGE model. As other recent New Keynesian models do, it embeds nominal rigidities in the form of Calvo (1983) wage and price contracts, habit formation in consumption, investment adjustment costs, variable capital utilization, and monetary policy governed by a Taylor rule.

However, relative to existing medium-scale macro models (Christiano et al., 2005; Smets and Wouters, 2007; Justiniano and Primiceri, 2008; Justiniano, Primiceri and Tambalotti, 2010, 2011), ours makes the following theoretical additions. A first addition is non-zero steady-state inflation. A second is real per capita output growth stemming from two distinct sources: trend growth in investment-specific technology (IST) and in neutral technology. Greenwood, Hercowitz, and Krusell (1997) show that investment-specific technological change has been a major source of U.S. economic growth during the postwar period. In our model, trend growth in IST realistically captures the downward secular movement in the relative price of investment observed during the postwar period. A third addition is roundabout production (Basu, 1995; Huang et al., 2004), an ingredient Christiano (2015) refers to as “firms networking” after Acemoglu, Akcigit and Kerr (2015). Evidence supporting roundabout production is discussed in Basu (1995), Huang et al. (2004) and Nakamura and Steinsson (2010). Roundabout production is also corroborated by a recent dataset gathered through the joint efforts of the NBER and the U.S. Census Bureau’s CES covering 473 six-digit 1997 NAICS industries for the years 1959-2009. A fourth addition is an extended working capital channel. Working capital has been a key feature of several macro models (Fuerst, 1992; Christiano, Eichenbaum and Evans, 1997, 2005; Barth and Ramey, 2002). We follow the approach in Phaneuf, Sims and Victor (2015) who assume that firms need working capital in advance of production to cover the costs of all of their variable inputs and not only the wage bill. They show that extended borrowing has several attractive implications such as helping a New Keynesian model with purely forward-looking wage and price setters to be consistent with a highly persistent and hump-shaped response of inflation, the possibility of a short-run price puzzle (Sims, 1991; Christiano et al. 1999, 2005), and a procyclical price markup at the onset of a monetary policy shock consistent with the evidence in Nekarda and Ramey (2013).

A notable difference with many New Keynesian models recently found in the literature, however, is that we abstract in our baseline model from the assumption that non reoptimized nominal wages and prices are indexed either fully or partially to the previous quarter’s rate of inflation and/or to steady-state inflation. The use of indexation has been criticized by a number of researchers. Woodford (2007), for instance, argues that “the model’s implication that prices should continuously
adjust to changes in prices elsewhere in the economy flies in the face of the survey evidence.”

Cogley and Sbordone (2008) mention that backward indexation “lacks a convincing microeconomic
foundation.” Chari et al. (2009) state that “this feature is inconsistent with microeconomic evidence
on price setting.” Finally, Christiano (2015) argues that the “no-indexation assumption is suggested
by the same microeconomic observations that motivate price setting frictions in the first place.
Those observations show that many prices remain unchanged for extended periods of time.”

The subsections below lay out the decision problems, while the optimality conditions of the
relevant model agents are in the Appendix.

2.1 Good and Labor Composites

There is a continuum of firms, indexed by \( j \in [0, 1] \), who produce differentiated goods with the
use of a composite labor input. The composite labor input is aggregated from differentiated labor
supplied by a continuum of households, indexed by \( i \in [0, 1] \). The differentiated goods are bundled
into a gross output good, \( X_t \). As we discuss below, some of this gross output good is used as a
factor of production by firms. Net output is therefore measured as gross output less intermediates.
The households can either consume or invest the final net output good. The composite gross output
and labor input respectively are:

\[
X_t = \left( \int_0^1 X_t(j) \frac{\theta - 1}{\theta} dj \right)^{\frac{\theta}{\theta - 1}},
\]

\[
L_t = \left( \int_0^1 L_t(i) \frac{\sigma - 1}{\sigma} di \right)^{\frac{\sigma}{\sigma - 1}}.
\]

The parameters \( \theta > 1 \) and \( \sigma > 1 \) are the elasticities of substitution between goods and labor.
The demand curves for goods and labor are:

\[
X_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} X_t, \quad \forall j,
\]

\[
L_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\sigma} L_t, \quad \forall i.
\]

The aggregate price and wage indexes are:

\[
P_t^{1-\theta} = \int_0^1 P_t(j)^{1-\theta} dj,
\]

\[
W_t^{1-\sigma} = \int_0^1 W_t(i)^{1-\sigma} di.
\]
2.2 Households

There is a continuum of households, indexed by $i \in [0, 1]$, who are monopoly suppliers of labor. They face a downward-sloping demand curve for their particular type of labor given in (4). Following Calvo (1983), each period there is a fixed probability, $(1 - \xi_w)$, that households can adjust their nominal wage. As in Erceg, Henderson, and Levin (2000), we assume that utility is separable in consumption and labor. State-contingent securities insure households against idiosyncratic wage risk arising from staggered wage-setting. With this setup, households will be identical along all dimensions other than labor supply and wages.

The problem of a typical household, omitting dependence on $i$ except for these two dimensions, is:

$$\max_{C_t, L_t(i), K_{t+1}, B_{t+1}, I_t, Z_t} \sum_{t=0}^{\infty} \beta^t \left( \ln (C_t - bC_{t-1}) - \eta L_t(i)^{1+\chi} \right),$$

subject to the following budget constraint,

$$P_t \left( C_t + I_t + \frac{a(Z_t)K_t}{\varepsilon_l^I I_{t-1}^{\tau}} \right) + \frac{B_{t+1}}{1 + i_t} \leq W_t(i)L_t(i) + R^k_t Z_t K_t + \Pi_t + B_t + T_t,$$

and the physical capital accumulation process,

$$K_{t+1} = \vartheta_t (1 - S(I_t I_{t-1})) I_t + (1 - \delta) K_t.$$

Here, $P_t$ is the nominal price of goods, $C_t$ is consumption, $I_t$ is investment measured in units of consumption, $K_t$ is the physical capital stock, and $Z_t$ is the level of capital utilization. $W_t(i)$ is the nominal wage paid to labor of type $i$, and $R^k_t$ is the common rental price on capital services (the product of utilization and physical capital). $\Pi_t$ and $T_t$ are, respectively, distributed dividends from firms and lump sum taxes from the government, both of which households take as given. $B_t$ is a stock of nominal bonds that the household enters the period with. $a(Z_t)$ is a resource cost of utilization, satisfying $a(1) = 0$, $a'(1) = 0$, and $a''(1) > 0$. This resource cost is measured in units of physical capital. $S \left( \frac{I_t}{I_{t-1}} \right)$ is an investment adjustment cost, satisfying $S(g_I) = 0$, $S'(g_I) = 0$, and $S''(g_I) > 0$, where $g_I \geq 1$ is the steady state (gross) growth rate of investment. $i_t$ is the nominal interest rate. $0 < \beta < 1$ is a discount factor, $0 < \delta < 1$ is a depreciation rate, and $0 \leq b < 1$ is a parameter for internal habit formation. $\chi$ is the inverse Frisch labor supply elasticity. $\varepsilon_{t+1}^{I, \tau}$, which enters the budget constraint in terms of the resource cost of capital utilization and the relative price of investment to consumption goods, measures the level of IST. We assume that

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5 The relative price of investment goods to consumption goods is $\frac{1}{\varepsilon_{I_t}}$. Hence, if $\hat{I}_t$ is physical units of investment, then $\varepsilon_{t}^{I} I_t = \hat{I}_t$. Writing the accumulation equation in terms of investment measured in consumption units yields (9).
it follows a deterministic trend with no stochastic component. The deterministic trend is necessary to match the observed downward trend in the relative price of investment goods in the data. The exogenous variable $\theta_t$, which enters the capital accumulation equation in the same way as the IST term, is a stochastic MEI shock.

Justiniano, Primiceri, and Tambalotti (2011) distinguish between these two types of investment shocks, showing that IST shocks map one-to-one into the relative price of investment goods, while MEI shocks do not impact the relative price of investment.\(^6\) They find that MEI shocks are critical for business cycles, while stochastic shocks to IST virtually have no effect on output at business cycle frequencies. These findings form the basis for our modeling choice of having the MEI component stochastic while the IST term only affects trend growth.

A household given the opportunity to adjust its wage in period $t$ will choose a “reset wage” to maximize the expected value of discounted flow utility, where discounting in period $t+s$ is $(\beta \xi w)^s$, $\xi_s$ being the probability that a wage chosen in period $t$ will still be in effect in period $t+s$. Given our assumption on preferences and wage-setting, all updating households will choose the same reset wage, denoted in real terms by $w_t^*$. The optimal reset wage is given by:

$$w_t^* = \frac{\sigma}{\sigma - 1} \frac{f_{1,t}}{f_{2,t}}.$$  \hspace{1cm} (10)

The terms $f_{1,t}$ and $f_{2,t}$ can be written recursively as:

$$f_{1,t} = \eta \left( \frac{w_t}{w_t^*} \right)^{\sigma(1+\chi)} L_t^{1+\chi} + \beta \xi_w E_t(\pi_{t+1})^{\sigma(1+\chi)} \left( \frac{w_{t+1}^*}{w_t^*} \right)^{\sigma(1+\chi)} f_{1,t+1}$$ \hspace{1cm} (11)

and

$$f_{2,t} = \lambda_t \left( \frac{w_t}{w_t^*} \right)^{\sigma} L_t + \beta \xi_w E_t(\pi_{t+1})^{\sigma-1} \left( \frac{w_{t+1}^*}{w_t^*} \right)^{\sigma} f_{2,t+1}. \hspace{1cm} (12)$$

2.3 Firms

The production function for a typical producer $j$ is:

$$X_t(j) = \max \left\{ A_t \Gamma_t(j)^{\phi} \left( \bar{K}_t(j)^{\alpha} L_t(j)^{1-\alpha} \right)^{1-\phi} - \Upsilon_t F, 0 \right\},$$ \hspace{1cm} (13)

where $F$ is a fixed cost, and production is required to be non-negative. $\Upsilon_t$ is a growth factor, to be discussed later. Given $\Upsilon_t$, $F$ is chosen to keep profits zero along a balanced growth path, so the

\(^6\)In the model, the relative price of investment goods is easily seen to be $\frac{1}{\varepsilon I,\tau}$. The division by $\varepsilon I,\tau$ in the resource cost of utilization is therefore necessary so that capital is priced in terms of consumption goods.
entry and exit of firms can be ignored. $\Gamma_t(j)$ is the amount of intermediate input, and $\phi \in (0, 1)$ is the intermediate input share. Intermediate inputs come from aggregate gross output, $X_t$. $\hat{K}_t(j)$ is capital services (the product of utilization and physical capital), while $L_t(j)$ is labor input. This production function differs from the standard in the New Keynesian DSGE literature in its addition of intermediate goods, $\Gamma_t(j)$.

The firm gets to choose its price, $P_t(j)$, as well as quantities of intermediates, capital services, and labor input. It is subject to Calvo (1983) pricing, where each period there is a $(1 - \xi_p)$ probability that a firm can re-optimize its price. Regardless of whether a firm is given the opportunity to adjust its price, it will choose inputs to minimize total cost, subject to the constraint of producing enough to meet demand. The cost minimization problem of a typical firm is:

$$\min_{\Gamma_t, \hat{K}_t, L_t} \left(1 - \psi_T + \psi_T(1 + i_t)\right)P_t \Gamma_t + (1 - \psi_K + \psi_K(1 + i_t))R_k^k\hat{K}_t + (1 - \psi_L + \psi_L(1 + i_t))W_t L_t$$

s.t.

$$\Gamma_t^\phi \left(\hat{K}_t^\alpha L_t^{1-\alpha}\right)^{1-\phi} - F \geq \left(\frac{P_t(j)}{P_t}\right)^{-\theta} X_t.$$  \hspace{1cm} (15)

Here $\psi_l$, $l = \Gamma, K, L$, is the fraction of payments to a factor that must be financed at the gross nominal interest rate, $1 + i_t$. Assuming $\psi_l = 1$ for all $l$ means that all factor payments are financed through working capital, so that the factor prices relevant for firms are the product of the gross nominal interest rate and the factor price. We refer to this case as extended borrowing (EB). With $\psi_l = 0$ for all $l$, firms do not have to borrow to pay any of their factors.\(^7\) To economize on notation, we define $\Psi_t = (1 - \psi_l + \psi_l(1 + i_t))$ for $l = \Gamma, K, L$.

Assume that $\psi_l = 1$ for $l = \Gamma, K, L$. Applying some algebraic manipulations to the first order conditions for cost-minimization yields the following expression for real marginal cost, $v_t$, which is common across firms:

$$v_t = (1 + i_t) \left(\frac{1}{1 - \phi}\right)^{-\phi} \left(\frac{1}{\phi}\right)^\phi \tilde{v}_t^{1-\phi},$$

where $\tilde{v}_t$ is the standard real marginal cost given a Cobb-Douglas production function without roundabout production ($\phi = 0$) and extended borrowing, that is:

\(^7\)Setting $\psi_T = \psi_K = 0$ and $\psi_L = 1$ would mean that firms make use of working capital to finance their wage bill (Christiano, Eichenbaum, and Evans, 1997, 2005; Ravenna and Walsh, 2006).
Relative to the basic case in the literature, roundabout production reduces the sensitivity of real marginal cost to factor prices by a factor of $1 - \phi$. Second, the nominal interest rate is a direct component of real marginal cost.

When given the opportunity to adjust its price, a firm will maximize the expected discounted value of profits. Discounting in period $t + s$ is by the stochastic discount factor as well as $\xi_p^s$, $\xi_p^s$ being the probability that a price chosen in period $t$ will still be in effect in period $t + s$. All updating firms will choose the same reset price. Let $p_t^* = \frac{P_t^*}{P_t}$ be the optimal reset price relative to the aggregate price index. The optimal pricing condition can be written:

$$p_t^* = \frac{\theta}{\theta - 1} \frac{x_{1,t}}{x_{2,t}}. \quad (18)$$

The auxiliary variables $x_{1,t}$ and $x_{2,t}$ can be written recursively:

$$x_{1,t} = \lambda r_t v_t X_t + \beta \xi_p E_t(\pi_{t+1})^\theta x_{1,t+1}, \quad (19)$$

$$x_{2,t} = \lambda r_t X_t + \beta \xi_p E_t(\pi_{t+1})^{\theta-1} x_{1,t+1}. \quad (20)$$

In these expressions $\lambda r_t$ is the marginal utility of an additional unit of real income received by the household. $X_t$ is aggregate gross output.

### 2.4 Monetary Policy

Monetary policy follows a Taylor rule:

$$\frac{1 + i_t}{1 + i} = \left( \frac{1 + i_{t-1}}{1 + i} \right)^{\rho_i} \left( \frac{\pi_t}{\pi} \right)^{\alpha_\pi} \left( \frac{Y_t}{Y_{t-1}} - g_Y^{-1} \right)^{\alpha_y} \varepsilon_t^r. \quad (21)$$

The nominal interest rate responds to deviations of inflation from an exogenous steady-state target, $\pi$, and to deviations of output growth from its trend level, $g_Y$. $\varepsilon_t^r$ is an exogenous shock to the policy rule. The parameter $\rho_i$ governs the smoothing-effect on nominal interest rates while $\alpha_\pi$ and $\alpha_y$ are control parameters.
2.5 Shock Processes

Neutral productivity obeys a process with both a trending and stationary component. $A^*_t$ is the deterministic trend component, where $g_A$ is the gross growth rate:

$$A_t = A^*_t \bar{A}_t, \quad (22)$$

$$A^*_t = g_A A^*_{t-1}. \quad (23)$$

The initial level in period 0 is normalized to 1: $A^*_0 = 1$. The stationary component of neutral productivity follows an AR(1) process in the log, with the non-stochastic mean level normalized to unity, and innovation, $u^A_t$, drawn from a mean zero normal distribution with known standard deviation equal to $s_A$:

$$\bar{A}_t = (\bar{A}_{t-1})^{\rho_A} \exp(s_A u^A_t), \quad 0 \leq \rho_A < 1, \quad (24)$$

The IST term obeys the following deterministic trend, where $g_{\varepsilon I}$ is the gross growth rate and the initial level in period 0 is normalized to unity:

$$\varepsilon^I_{t,\tau} = g_{\varepsilon I} \varepsilon^I_{t-1} \quad (25)$$

The MEI shock follows a stationary AR(1) process, with innovation drawn from a mean zero normal distribution with standard deviation $s_I$:

$$\vartheta_t = (\vartheta_{t-1})^{\rho_I} \exp(s_I u^I_t), \quad 0 \leq \rho_I < 1 \quad (26)$$

The only remaining shock in the model is the monetary policy shock, $\varepsilon^r_t$. We assume that it is drawn from a mean zero normal distribution with known standard deviation $s_r$.

2.6 Functional Forms

We assume that the resource cost of utilization and the investment adjustment cost function have the following functional forms:

$$a(Z_t) = \gamma_1(Z_t - 1) + \frac{\gamma_2}{2}(Z_t - 1)^2, \quad (27)$$

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - g_I\right)^2, \quad (28)$$
where $\gamma_2 > 0$ is a free parameter; as $\gamma_2 \to \infty$ utilization becomes fixed at unity. $\gamma_1$ must be restricted so that the optimality conditions are consistent with the normalization of steady state utilization of 1. $\kappa \geq 0$ is a free parameter. The functional form for the investment adjustment cost is standard in the literature (e.g. see Christiano, Eichenbaum, and Evans, 2005).

### 2.7 Growth

Most variables in the model will inherit trend growth from the deterministic trends in neutral and investment-specific productivity. Let this trend factor be $\Upsilon_t$. Output, consumption, investment, intermediate inputs, and the real wage will all grow at the rate of this trend factor on a balanced growth path: $g_Y = g_I = g_r = g_w = g_\Upsilon$. The capital stock will grow faster due to growth in investment-specific productivity, with $\tilde{K}_t \equiv \frac{K_t}{\Upsilon_t}$ being stationary. Given our specification of preferences, labor hours will be stationary. The full set of equilibrium conditions re-written in stationary terms can be found in the Appendix.

One can show that the trend factor that induces stationarity among transformed variables is:

$$\Upsilon_t = (A_t^\tau)^{\frac{1}{1-\phi}} \left( \varepsilon_t^{I,\tau} \right)^{\frac{\phi}{1-\phi}}.$$

(29)

This reverts to the conventional trend growth factor in a model with growth in neutral and investment-specific productivity when $\phi = 0$. Under this restriction, intermediates are irrelevant for production, and the model reduces to the standard New Keynesian model. Interestingly, from (29), it is evident that a higher value of $\phi$ amplifies the effects of trend growth in neutral productivity on output and its components. For a given level of trend growth in neutral productivity, the economy will grow faster the larger is the share of intermediates in production.

### 3 Calibration

We split the baseline calibration of the model’s parameters in two groups: non-shock and shock parameters.

#### 3.1 Non-Shock Parameters

The values of non-shock parameters are summarized in Table 1. $\beta = 0.99$ is the discount factor, $b = 0.8$ is the habit formation parameter, $\chi = 1$ is the inverse Frisch elasticity, and $\eta = 6$ is the weight on disutility of labor set so that steady-state labor hours are around 1/3.
The parameters $\theta$ and $\sigma$ are the elasticities for goods and labor which are both set at 6 (Rotemberg and Woodford, 1997, and Liu and Phaneuf, 2007). The Calvo price and wage probabilities, $\xi_p$ and $\xi_w$, are set at $2/3$. Using a dataset covering the frequency of price changes for 350 categories of consumer goods and services for the years 1995-1997, Bils and Klenow (2004) find that the median duration of U.S. prices ranges between 4.3 and 5.5 months. Cogley and Sbordone (2008) link the median duration of prices to the Calvo probability of price non-reoptimization by $-\ln(2)/\ln(\xi_p)$. Setting $\xi_p = 2/3$ therefore implies a median duration of prices of 5 months, which is broadly consistent with the evidence presented by Bils and Klenow. By setting $\xi_w = 2/3$, we adopt a conservative stance. While this value is broadly consistent with the macro estimate reported by Christiano, Eichenbaum, and Evans (2005), it is somewhat lower than the micro evidence offered by Barattieri, Basu, and Gottschalk (2014), and also somewhat lower than the estimates in Justiniano, Primiceri, and Tambalotti (2010, 2011).

The parameters in the production function are the share of capital services $\alpha = 1/3$ and the share of intermediate inputs $\phi = 0.61$. The value of $\alpha$ is standard. The parameter $\phi$ is obtained as follows. As in Nakamura and Steinsson (2010), the weighted average revenue share of intermediate inputs in the U.S. private sector using Consumer Price Index (CPI) expenditure weights is roughly 51% in 2002. The cost share of intermediate inputs is equal to the revenue share times the markup. Our calibration of $\theta$ implies a markup of 1.2. Therefore, our estimate of the weighted average cost share of intermediate inputs is roughly 61%.\(^8\)

The parameter $\delta = 0.025$ is the depreciation rate on physical capital, $\kappa = 3$ is the investment adjustment cost parameter consistent with the estimates reported in Christiano, Eichenbaum, and Evans (2005) and Justiniano, Primiceri, and Tambalotti (2010). $\gamma_1$ is set so that steady state utilization is 1. The parameter $\gamma_2$ is set to be equal to five times $\gamma_1$, consistent with the structural estimates provided in Justiniano, Primiceri, and Tambalotti (2010, 2011).

We assume that firms have to fully finance the costs of all variable inputs of production. That is, we assume that $\psi_L = \psi_K = \psi_T = 1$. This is based on the analysis in Phaneuf, Sims, and Victor (2015), who show that this form of extended borrowing can help models generate hump-shaped inflation dynamics and a price puzzle conditional on a monetary policy shock without relying on backward price and wage indexation. Christiano, Eichenbaum, and Evans (1997, 2005) and Ravenna and Walsh (2006) consider the case where firms have to borrow to finance only the wage bill, robustness to our parameterization along this dimension.

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\(^8\)The steady-state price markup is for a trend inflation of zero. We find that this markup is almost insensitive to trend inflation between 0 and 4 percent leaving $\phi$ unaffected as trend inflation rises.
The last three parameters are for the Taylor rule, and include the smoothing parameter set at 0.8, the coefficient on inflation at 1.5, and the coefficient on output growth at 0.2. These values are fairly standard in the literature.

3.2 Trend Inflation and Trend Growth

We now explain the calibration of parameters governing trend inflation, and trend growth. Table 2 summarizes the values assigned to these parameters.

Mapping the model to the data, the trend growth rate of the IST term, $g_{\epsilon t}$, equals the negative of the growth rate of the relative price of investment goods. To measure this in the data, we define investment as expenditures on new durables plus private fixed investment, and consumption as consumer expenditures of nondurables and services. These series are from the BEA and cover the period 1960:I-2007:III.

Let $C^n_{nd,t}$, $C^n_{s,t}$, $D^n_t$, and $I^n_f$ denote nominal non-durable consumption, services consumption, expenditure on durables, and fixed investment. Let $P^n_{nd,t}$, $P^n_{s,t}$, $P^n_{d,t}$, and $P^n_{f,t}$ denote the corresponding price indexes. Nominal consumption and nominal investment are then:

$$C^n_t = C^n_{nd,t} + C^n_{s,t},$$

$$I^n_t = D^n_t + I^n_f.$$  \(30\) \(31\)

Let $g_{nd,t}$, $g_{s,t}$, $g_{d,t}$, and $g_{f,t}$ denote the real growth rates of the series:

$$g_{nd,t} = \ln C^n_{nd,t} - \ln C^n_{nd,t-1} - (\ln P^n_{nd,t} - \ln P^n_{nd,t-1}),$$

$$g_{s,t} = \ln C^n_{s,t} - \ln C^n_{s,t-1} - (\ln P^n_{s,t} - \ln P^n_{s,t-1}),$$

$$g_{d,t} = \ln C^n_{d,t} - \ln D^n_{t,t-1} - (\ln P^n_{d,t} - \ln P^n_{d,t-1}),$$

$$g_{f,t} = \ln I^n_{f,t} - \ln I^n_{f,t-1} - (\ln P^n_{f,t} - \ln P^n_{f,t-1}).$$  \(32\) \(33\) \(34\) \(35\)

The real growth rate of non-durable and services consumption is the share-weighted growth rates of the real component series:

$$g_{c,t} = \left(\frac{C^n_{nd,t-1}}{C^n_{t-1}}\right) g_{nd,t} + \left(\frac{C^n_{s,t-1}}{C^n_{t-1}}\right) g_{s,t}.$$

\(36\)
The real growth rate of investment is the share-weighted growth rates of the real components:

\[ g_{i,t} = \left( \frac{D^n_{t-1}}{I^n_{t-1}} \right) g_{d,t} + \left( \frac{I^n_{f,t-1}}{I^n_{t-1}} \right) g_{f,t}. \]  

(37)

The log-level real series is computed by cumulating the growth rates starting from a base of 1. To put them in levels, we exponentiate the log-levels. Then they are re-scaled so that the real and nominal series are equal in the third quarter of 2009. The price indexes for consumption and investment are computed as the ratios of the nominal to the real series. The relative price of investment is the ratio of the implied price index for investment goods to the price index for consumption goods. The average growth rate of the relative price from the period 1960:I-2007:III is -0.00472. This implies a calibration of \( g_{\varepsilon I} = 1.00472. \)

We compute aggregate output in a similar way. Define nominal output as the sum of the nominal components:

\[ Y^n_t = C^n_{nd,t} + C^n_{s,t} + D^n_t + I^n_{f,t}. \]  

(38)

The growth rate of real GDP is calculated by using the share-weighted real growth rates of the constituent series:

\[ g_{y,t} = \left( \frac{C^n_{nd,t-1}}{Y^n_{t-1}} \right) g_{nd,t} + \left( \frac{C^n_{s,t-1}}{Y^n_{t-1}} \right) g_{s,t} + \left( \frac{D^n_{t-1}}{Y^n_{t-1}} \right) g_{d,t} + \left( \frac{I^n_{f,t-1}}{Y^n_{t-1}} \right) g_{f,t}. \]  

(39)

Then, we cumulate to get in log-levels, and exponentiate to get in levels. The price deflator is obtained as the ratio between the nominal and real series. The average growth rate of the price index over the period 1960:I-2007:III is 0.008675. This implies \( \pi^* = 1.0088 \) or 3.52 percent annualized.

Real per capita GDP is computed by subtracting the log civilian non-institutionalized population from the log-level of real GDP. The average growth rate of the resulting output per capita series over the period is 0.005712. The standard deviation of output growth over the period is 0.0078. The calculations above imply that \( g_y = 1.005712 \) or 2.28 percent a year. Given the calibrated growth of IST from the relative price of investment data \( (g_{\varepsilon I} = 1.00472) \), we then pick \( g_A^{1-\phi} \) to generate the appropriate average growth rate of output. This implies \( g_A^{1-\phi} = 1.0022 \) or a measured growth rate of TFP of about 1 percent per year.\(^9\)

\(^9\)Note that this is a lower average growth rate of TFP than would obtain under traditional growth accounting exercises. This is due to the fact that our model includes roundabout production, which would mean that a traditional growth accounting exercise ought to overstate the growth rate of true TFP.
3.3 Shock Parameters

We next turn to the parameterization of the shock processes in our model. These parameters are summarized in Table 2. The baseline model includes three types of shocks: neutral productivity, marginal efficiency of investment, and monetary policy. In Christiano, Eichenbaum, and Evans (2005), fluctuations are driven only by shocks to monetary policy. In Smets and Wouters (2007), in contrast, there are seven different shocks. Some papers in the literature recently questioned the increasing number of disturbances in recent models. For example, referring to Smets and Wouters (2007), Chari, Kehoe, and McGrattan (2009) argue that only three – the productivity shock, the investment shock, and the monetary policy shock – can be considered structural. The other four shocks, which include shocks to preferences, the consumption Euler equation, and the aggregate accounting identity, are dubiously structural and do not have a clear economic interpretation.

Neutral productivity shocks are typically estimated to be quite persistent. This finding emerges both in structural Bayesian estimations of fully-specified DSGE models as well as in univariate growth accounting exercises. Accordingly, we set the autoregressive parameter of the neutral productivity shock at $0.95$. There is less compelling evidence on the persistence of the marginal efficiency of investment shock. We follow Justiniano, Primiceri, and Tambalotti (2011) and set the autoregressive parameter of the MEI process at $0.81$. We later assess sensitivity of our results for higher or lower values of this parameter, which ends up being crucial for the cyclical implications of trend inflation.

To pin down the standard deviations of three shocks in our model, we proceed as follows. We target a size of shocks $s_I$, $s_A$, and $s_r$, for which our baseline model exactly matches the actual volatility of output growth observed in our data ($0.0078$) for a quarterly average trend inflation equal to its observed value during the postwar period ($\pi^* = 1.0088$). To determine the exact numbers for $s_I$, $s_A$, and $s_r$, we assign to each type of shock a target percentage contribution to the unconditional variance decomposition of output growth. In particular, target a 50 percent share of the variance of output growth due to the MEI shock, 35 percent to the productivity shock, and 15 percent to the monetary shock. This implies values of $s_I = 0.0276$, $s_A = 0.0030$, and $s_r = 0.0020$.

Our targets for the contribution of the three shocks to the variance of output growth are based on empirical consensus from the recent literature. In this literature, investment shocks are the main driver behind business-cycle fluctuations, followed by neutral technology shocks. In the estimates from both Justiniano, Primiceri, and Tambalotti (2010), the investment shock explains about 50 percent of the variance decomposition of output growth at business cycle frequencies, followed by the neutral technology shock with 25 percent and by the monetary policy shock with 5 percent. This
leaves only 20 percent to be explained by other types of shocks (that is, by government-spending, price-markup, wage-markup and preference shocks). In Justiniano et al. (2011), a distinction is drawn between an investment-specific technology (IST) shock and a shock to the marginal efficiency of investment (MEI). The MEI shock explains 60 percent of fluctuations in output growth, the neutral technology 25 percent, the monetary policy shock 4 percent and the IST shock 0 percent. This leaves only 11 percent of output fluctuations to be explained by other types of shocks. Some other studies in which investment shocks explain a larger fraction of output fluctuations than TFP shocks include Fisher (2006), Justiniano and Primiceri (2008) and Altig, Christiano, Eichenbaum and Lindé (2011).

One exception is Smets and Wouters (2007), who report that investment shocks account for less than 25 percent of the forecast error variance of GDP at any horizon. Justiniano et al. (2010) explore the reasons behind these differences in results, showing that the smaller contribution of investment shocks in SW is due essentially to their unusual definition of consumption and investment which includes durable expenditures in consumption while excluding the change in inventories from investment, although not from output. With the more standard definition of consumption and investment found in the business-cycle literature (Cooley and Prescott, 1995; Christiano, Eichenbaum, and Evans, 2005; Del Negro, Schorfheide, Smets and Wouters, 2007), investment shocks are the main drivers behind business-cycle fluctuations, and this by a good margin.

3.4 Selected Moments

To assess the empirical relevance of our baseline model, we analyze some basic business cycle moments and compare them to moments from the data. The model is solved via second order perturbation about the non-stochastic steady state. The moments are summarized in Table 3. The reported volatility and correlation statistics are for variables measured in growth rates or as deviations from stochastic trends obtained using the HP filter.

The mean value of real per capita output growth implied by the model, $E(\Delta Y)$, matches the data at 0.0057, or 2.28 percent annualized. The volatility of output growth equals the actual volatility by construction. The model slightly overpredicts the volatility of HP-filtered log output relative to the data. The model does a very good job matching the volatility of consumption in the data, whether measured in growth rate or HP filtered log-levels. The volatility of investment is somewhat overestimated by our model, but remains plausible. The volatility of first-differenced hours is somewhat higher in the model than in the data, while the model slightly underestimates the volatility of HP-filtered log hours. The baseline model somewhat underestimates the variability of inflation in the data (0.0044 vs 0.0064).
The correlation between the growth rates of consumption and output predicted by our baseline model fits the data quite well (0.63 vs 0.75), while our model slightly underpredicts the cyclicality of consumption with output when measured in HP filtered log levels (0.59 versus 0.91). The cyclicality of consumption in our model is higher, and hence more in-line with the data, with respect to the one reported in Justiniano et al. (2010, 2011). As we later argue, the stronger comovement between consumption and output can be explained by the fact that our model is able to avoid a short-run decline in consumption following a positive MEI shock. It is well known that most DSGE models predict that consumption initially falls at the onset of a positive MEI shock. We return to this point later. Our model accurately predicts that the correlation between investment and output is very high, whether measured in growth rates or filtered log-levels. Similarly, the model predicts that labor hours are strongly procyclical, measured in either growth rates or HP filtered log-levels, though the model slightly underpredicts the overall cyclicality of hours.

The first-order autocorrelation of inflation predicted by the model is high at 0.82.\textsuperscript{10} Note that our baseline model predicts that inflation is highly persistent in spite of the fact that it abstracts from wage and price indexation to past inflation. The model also generates a positive first-order autocorrelation of output growth at 0.65 compared to 0.36 in the data, which according to Cogley and Nason (1995) is a useful test of the strength of the endogenous business-cycle propagation mechanisms embedded in a particular model. Overall, the baseline model performs quite well along several usual business-cycle dimensions.

4 The Welfare Costs of Trend Inflation

This section examines the normative implications of moderate trend inflation. Our analysis focuses on the following two statistics: \(i\) a consumption-equivalent welfare loss metric denoted \(\lambda_{ss}\) conditioned on steady states and measuring how much consumption needs to be taken away in a low inflation state for households to have the same welfare as in a high inflation state, and \(ii\) an equivalent metric denoted \(\lambda_m\) and conveying information conditioned on stochastic means.\textsuperscript{11} Table 4 reports the welfare costs implied by the benchmark model, with panel \(a\) showing welfare losses computed from non-stochastic steady states while panel \(b\) presents welfare losses computed from stochastic means. This table focuses on various different changes in the inflation target, though in

\textsuperscript{10}The autocorrelations of inflation implied by the model are positive at lags of 1 to 6 quarters (not reported).

\textsuperscript{11}To do this we include a recursive measure of aggregate welfare as an equilibrium condition and compute its mean value under different levels of trend inflation after solving the model via a second order approximation, as in Schmitt-Grohé and Uribe (2004).
the text we focus mostly on the costs of going from a two to four percent (annualized) target, a scenario consistent with many recent policy proposals.

According to our benchmark model, increasing trend inflation from 2 to 4 percent would generate a consumption-equivalent welfare loss of 3.73 percent conditioned on non-stochastic steady states and 4.3 percent conditioned on stochastic means. The gap between the welfare loss based on the stochastic mean and the one based on the deterministic steady state depends on the properties of the stochastic processes as well as other features of the model, points to which we return in more depth below. Based on stochastic means, the cost of going from 2 to 4 percent is nearly twice as large. This non-linearity is important when thinking about policies to raise the inflation target in light of the zero lower bound, as in Coibion, Gorodnichenko, and Wieland (2012). While very small amounts of trend inflation might be desirable to reduce the frequency of ZLB episodes, going from 2 to 4 percent trend inflation would result in substantially larger welfare costs.

Our metrics for the welfare loss of trend inflation are larger than most reported values in the existing literature. While extremely high levels of trend inflation can imply large costs, it is generally found that modest amounts of trend inflation (say, between 0 and 4 percent annualized) have small welfare costs. One exception to the literature which finds small costs of moderate trend inflation is Amano et al. (2009), who find that increasing trend inflation from its optimal level (slightly negative in their model) to 4 percent results in a mean welfare cost of about two percent of consumption. The welfare costs we find are almost three times larger than that – in our model, going from 0 percent trend inflation to 4 percent implies a welfare cost of about 6.5 percent based on stochastic means. Our model shares two important features with theirs – the coexistence of both price and wage rigidity as well as trend output growth – but includes several features from which their model abstracts. In addition to capital accumulation which is absent in their model, our model also features a number of real rigidities and frictions. Some of these are relatively standard in the literature – habit formation in consumption, variable capital utilization, and investment adjustment costs – while some others are not as common. These features include extended borrowing, roundabout production, and important stochastic shocks to the marginal efficiency of investment. We discuss in turn the roles played by all of these features in driving our results.

Table 5 shows both the steady state and mean welfare losses from going from 2 to 4 percent (annualized) trend inflation in a variety of different specifications. Deviations from our benchmark

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12See Amano, Ambler, and Rebei (2007) for an analysis of how trend inflation affects the steady-state and mean values of key macroeconomic variables. That the welfare loss based on stochastic means is higher than the one based on the deterministic steady state is consistent with their analysis.
specification are described in the left column. When changing any feature of the model, the standard deviations of the shocks are re-calibrated to match the observed volatility of output growth as well as our specified variance decomposition. This is done to facilitate comparisons with our benchmark case. The first row considers the case where wages are flexible, $\xi_w = 0$. Here the welfare costs of trend inflation are substantially smaller than in our benchmark analysis – the mean cost of going from 2 to 4 percent trend inflation is only about 1 percent of consumption. The next row considers the case of no trend growth. For this exercise we set the trend growth rates of both the IST and neutral productivity terms to zero. Here the mean cost of going from two to four percent trend inflation is about 2.5 percent of consumption, also substantially smaller than our benchmark results. It does not make much difference whether trend growth comes from neutral productivity or investment specific technical change. In row (iii) we consider the case in which trend growth in output comes exclusively from neutral productivity, and in row (iv) the one in which trend growth comes exclusively from IST. The steady state and mean welfare costs of higher trend inflation are about the same in both cases as in our benchmark specification where trend growth arises from both sources. Row (v) considers the case where there is no trend growth in output and flexible wages. The welfare costs in this case are similar to the case when there is trend growth but wages are flexible. Row (vi) presents welfare costs of higher trend inflation when prices are flexible. Here the welfare costs of higher trend inflation based on the deterministic steady state are slightly smaller than in our baseline analysis, but the costs based on stochastic means are actually somewhat higher when prices are flexible. This arises due to the fact that we re-calibrate the shock sizes to match the observed volatility of output growth for each iteration of the model.

We next turn to an analysis of the role of real features in the model in accounting for the welfare costs of trend inflation. These results are summarized in Table 6, which is structured similarly to Table 5. We first consider the case in which there is no roundabout production, i.e. $\phi = 0$. This results in a mean welfare cost of going from 2 to 4 percent trend inflation of 3.7 percent, or about 0.6 percentage points lower than our baseline welfare cost. In row (ii) we consider the role of our assumption of extended borrowing, wherein firms must borrow to finance all variable inputs. The absence of this feature in the model leads to a similar reduction in the welfare cost of trend inflation as does roundabout production. In row (iii) we consider the case in which only the wage bill must be borrowed in advance, which is a fairly common assumption in the literature (for example, this is the assumed structure in Christiano, Eichenbaum, and Evans, 2005). Here the mean cost of increasing the trend inflation rate from 2 to 4 percent is 3.8 percent, substantially lower than our benchmark case.
Next, we consider the role of marginal efficiency of investment shocks, which in our calibration account for half of the variance of output growth. To our knowledge, no previous study has examined the interaction between trend inflation and MEI shocks. For this particular exercise, we set the variance of the MEI shock to zero and re-calibrate the magnitudes of the productivity and monetary policy shocks are to explain 75 and 25 percent of the variance of output growth, respectively. Although the absence of MEI shocks does not have any impact on the steady state costs of trend inflation, the consumption equivalent welfare loss based on stochastic means is about 0.4 percentage points lower than in our baseline analysis at 3.9 percent. When all three of the aforementioned features are excluded from the model – roundabout production, extended borrowing, and MEI shocks – the mean welfare cost of going from 2 to 4 percent trend inflation is a full percentage point lower than in our baseline, at 3.3 percent of consumption. The important conclusion of these exercises is that real rigidities and frictions, which are often included to give a model more realistic amplification and propagation, tend to magnify the welfare costs of trend inflation.

Panel (vi) considers the case where there is no roundabout production, no extended borrowing, and no MEI shocks, but in addition turns off wage rigidity and trend output growth. Here the welfare costs are very small, amounting to about 0.2 percent of consumption. Comparing these results to row (v) of Table 5, roundabout production, extended borrowing, and MEI shocks substantially amplify the welfare costs of trend inflation even without wage stickiness and trend growth – the welfare cost is about 1 percent with these features, or five times larger than when these features are absent and wages are flexible and there is no trend growth. Panel (vii) turns off the productivity shock but otherwise assumes the same parameterization and structure of our baseline model. For this exercise we parameterize the standard deviations of the MEI and monetary shocks to account for 75 and 25 percent of the variance of output growth, respectively. This alteration of our parameterization increases the mean welfare cost to 4.45 percent of consumption, or about 0.15 percent more than in our baseline. This exercise further confirms that the interaction between trend inflation and MEI shocks is relatively costly when compared with the two other shocks in our model.

We next consider some robustness of our welfare results to other key parameters. These results are presented in Table 7. Our analysis above suggests that wage rigidity is the key nominal friction giving rise to large welfare costs of trend inflation. Two key parameters governing how wage rigidity interacts with trend inflation are the Frisch labor supply elasticity and any indexation of wages to lagged inflation. In our model the inverse Frisch elasticity is given by the parameter $\chi$. Row (i) considers the case where this parameter is set to 0, implying an infinite Frisch elasticity, as in the indivisible labor models of Rogerson (1988) and Hansen (1985). Here we see that the welfare...
costs of trend inflation are substantially smaller than in our baseline analysis, with a cost of going from 2 to 4 percent trend inflation based on stochastic means of 1.5 percent. The intuition for the effect of $\chi$ on the welfare costs of trend inflation is straightforward. Trend inflation distorts the relative allocation of labor across households through an effect on wage dispersion. With curvature in preferences over labor, this misallocation can be quite costly. But if this curvature is absent, as is the case with $\chi = 0$ (or more generally relatively unimportant for low values of $\chi$), misallocated labor arising from wage dispersion has much smaller effects on welfare.

Panels (ii)-(iv) of Table 7 consider the role of wage indexation to past inflation. In particular, we assume that households not given the opportunity to adjust their wage in a period may nevertheless index their nominal wage to lagged inflation at rate $\gamma_w \in [0, 1]$. The case of $\gamma_w = 1$ is full indexation, which corresponds to the specification in Christiano, Eichenbaum, and Evans (2005), for example. When wages are fully indexed to lagged inflation, the mean welfare cost of going from 2 to 4 percent trend inflation is only about 1 percent, which is roughly in line with the results presented above when wages are fully flexible. Rows (iii)-(iv) consider the case of partial indexation, with values of $\gamma_w$ of 0.5 and 0.25. Here the welfare costs of trend inflation are substantially higher than the case of full wage indexation, but still lower than our benchmark specification which assumes no indexation. Price indexation has small impacts on our welfare results, which should not be surprising given that wage stickiness is the more important nominal rigidity. When prices are fully indexed to lagged inflation but wages are not, the mean welfare cost of going from 2 to 4 percent trend inflation is 3.8 percent of consumption.

The assumption of indexation mechanisms in New Keynesian models has attracted its share of criticism (Woodford, 2007; Cogley and Sbordone, 2008; and Chari, Kehoe, and McGrattan, 2009). One line of criticism is that backward indexation lacks microfoundations. Another is that it is at odds with the data since it implies that all wages and prices change every quarter (Bils and Klenow, 2004; Barattieri, Basu, and Gottschalk, 2014). DSGE models which estimate the degree of wage indexation typically find the indexing parameter is generally low, between 0 and 0.15 (Justiniano and Primiceri, 2008; Justiniano, Primiceri, and Tambalotti, 2010; Justiniano, Primiceri, and Tambalotti, 2011). Our baseline assumption of no indexation is in line with these findings.

5 The Cyclical Effects of Trend Inflation

This section analyzes the positive implications of moderate trend inflation. We focus on impulse responses of key macroeconomic variables to the three shocks in the model for different levels of
trend inflation. An important objective of this section is to highlight the strong but heretofore unknown (to our knowledge) interaction between MEI shocks and the level of trend inflation.

Figure 1 plots impulse responses of output and inflation to the three shocks in the model for three different levels of trend inflation. Solid lines are for the case of zero trend inflation, dashed lines for two percent (annualized) trend inflation, and dotted lines are for four percent trend inflation. There is not much of an effect of trend inflation on the responses of output and inflation to a productivity shock. For higher levels of trend inflation, the output response is slightly smaller at short forecast horizons and somewhat higher at longer horizons, but the effect is not very strong. Inflation falls by a little less for higher levels of trend inflation, but again the effect of trend inflation on the response is rather weak.

In our model output responds more to an expansionary monetary policy shock for higher levels of trend inflation. These effects are more noticeable in the graphs than for the case of the productivity shock. Interestingly, inflation jumps up by more on impact but is less persistent for higher levels of trend inflation. Note also that the response of inflation to the monetary shock is hump-shaped in spite of the fact that our baseline model features no price or wage indexation. This effect is largely driven by the model feature that payments to all factors must be borrowed, which is discussed in more detail in Phaneuf, Sims, and Victor (2015). While the interaction between trend inflation and the output response to a policy shock is qualitatively the same as in Ascari and Sbordone (2014), the effect on inflation is different. In their baseline New Keynesian model, inflation reacts less on impact but is more persistent for higher levels of trend inflation after a monetary policy shock. In our model the reverse is true.

We next turn attention to the interaction between trend inflation and the impulse responses to a MEI shock. Visually it is clear that the interaction between trend inflation and the MEI shock is stronger than for either the monetary or productivity shocks. Higher levels of trend inflation are associated with a larger output response to the MEI shock at all forecast horizons. The effect of trend inflation is largest at long forecast horizons. At a twenty quarter forecast horizon, for example, the output response is more than twice as large with four percent trend inflation than it is with zero trend inflation. There is an impact of trend inflation on the inflation response to the MEI shock, but it is not nearly as noticeable as with the output response. In particular, inflation reacts more strongly to the MEI shock for higher levels of trend inflation.

In the literature on investment-specific technology (IST) and marginal efficiency of investment (MEI) shocks (see Fisher, 2006 and Justiniano, Primiceri and Tambalotti, 2010 and 2011), an important issue is whether these shocks can generate a simultaneous increase in consumption, hours, and output. In standard flexible price neoclassical models consumption falls while investment and
output rise after a positive shock to the efficiency of transforming investment into new capital (the MEI shock) or consumption goods into investment goods (an IST shock). Figure 2 plots the impulse response of consumption to the MEI shock in our model for three different levels of trend inflation. Interestingly, our model is able to account for an increase in consumption to the MEI shock regardless of the level of trend inflation. Furthermore, as trend inflation increases, the consumption response to the MEI shock is larger at all forecast horizons. Justiniano, Primiceri and Tambalotti (2011) discuss how the interaction between monopolistic competition and price and wage rigidity can help resolve the co-movement problem between consumption and hours. Still, in their estimated model consumption initially declines after a positive MEI shock before taking about a year to rise. In our model consumption increases immediately after a MEI shock.

We briefly discuss the features of our model which can help generate a positive impact response of consumption to the MEI shock. Figure 3 shows impulse responses of consumption (for the same three levels of annualized trend inflation) with different features of our model turned off. The upper left panel shows the response of consumption when we turn off extended borrowing, roundabout production, and trend output growth. This specification of our model is essentially identical to Justiniano, Primiceri and Tambalotti (2010, 2011), and, unsurprisingly, the response of consumption (with zero trend inflation) is almost identical to what they report. Consumption (and output, though not shown) responds more strongly to the MEI shock for higher levels of trend inflation, as in our baseline model. The upper right panel turns off the financial intermediation channel, but keeps roundabout production and trend output growth. Once again, the response of consumption is stronger at all horizons for higher trend inflation, but is positive on impact for all three trend inflation rates considered. In fact, the response on impact here is larger than in our baseline case, suggesting that extended borrowing actually somewhat dampens the response of consumption to the MEI shock. The bottom two panels consider cases where roundabout production and trend output growth are shut off from our baseline model, respectively. In both cases the consumption response is stronger at all horizons for higher levels of trend inflation. The impact responses of consumption to the MEI shock are smaller in both cases than in our baseline, suggesting that these two features are important in generating the consumption increase after an MEI shock. The intuition for these effects is straightforward. Roundabout production effectively flattens the Phillips Curve, and therefore works to amplify the effects of demand shocks. Trend output growth makes inflation more forward-looking and hence less sensitive to current real marginal cost, which also has the effect of allowing demand shocks, like the MEI shock, to have stronger effects.

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13It should be noted that their model does feature trend output growth and trend inflation, but they assume an indexation structure which makes both trend growth and trend inflation irrelevant for equilibrium dynamics to a first order approximation.
We turn next to a discussion of the interaction between the level of trend inflation and the output response to a MEI shock. There is a strong and interesting relationship between the persistence of the shock and the effect of trend inflation. Figure 4 plots the impulse response of output to a MEI shock for three different levels of trend inflation. In the different panels we consider different values of $\rho_I$. For relatively small values of $\rho_I$, positive trend inflation results in a stronger output response to a MEI shock at all forecast horizons. But for higher values of $\rho_I$, the effect of trend inflation on the response to the MEI shock is the opposite. For $\rho_I = 0.95$, for example, the response of output to the MEI shock at four percent inflation is roughly one-third its value with zero trend inflation at medium forecast horizons. The shape of the response is also slightly different. For $\rho_I = 0.99$, these effects are exacerbated – the response of output to the MEI shock when trend inflation is four percent is negative for several years. In other words, it seems that at low levels of persistence higher rates of trend inflation make the MEI shock more expansionary, while the reverse is true at high levels of persistence in the shock.

To get a better sense of the relationship between the persistence of the MEI shock and the impact of trend inflation, Figure 5 plots the (absolute) difference of the output response to a MEI shock at a ten quarter horizon for inflation rates of four and two percent. This difference is increasing for values of $\rho_I$ up until about 0.75 and is positive for values of $\rho_I$ less than 0.9. For values of $\rho_I$ greater than 0.9, trend inflation exerts a negative effect on the response of output to a MEI shock. This effect is particularly large at values of $\rho_I$ near one.

The interaction between the persistence of the MEI shock, trend inflation, and the output response to the MEI shock suggests that the persistence of the shock might have important effects on the welfare costs of trend inflation. To examine this, we return to the exercises of the previous section. We begin by setting $\rho_I = 0.95$ and re-solving the model. Panel (viii) of Table 6 shows the mean welfare costs of increasing trend inflation from two to four percent with this higher persistence in the shock. The mean welfare cost is more than twice as large as under our baseline, with the cost of trend inflation amounting to more than 11 percent of consumption. In panel (ix), we consider the case in which the MEI shock is less persistent, with $\rho_I = 0.66$. Here the mean welfare cost of increasing trend inflation from two to four percent is 0.0396, or about 0.3 percentage points less than in our baseline. That trend inflation is relatively more costly from a welfare perspective for more persistent MEI shocks is entirely consistent with the impulse responses.

What is the intuition for the “sign flip” in the effect of trend inflation on the responses to MEI shocks for different levels of trend inflation? In the context of a textbook New Keynesian model with sticky prices only, Ascari (2004) shows that trend inflation both “flattens” the Phillips Curve and makes current inflation more sensitive to expected inflation. The MEI shock is an
aggregate demand shock, raising current demand for goods relative to supply and pushing output and inflation in the same direction. Holding expected inflation fixed (i.e. considering a sufficiently transitory shock so as to ignore any effect on expectations of the future), a flatter Phillips Curve due to higher trend inflation should imply that a positive demand shock ought to have a bigger effect on output. This is exactly what we observe for moderate levels of persistence in the MEI shock – output rises by more the higher is trend inflation. Holding the slope of the Phillips Curve fixed, a heightened sensitivity to the future due to higher trend inflation ought to result in a shock which raises expected inflation having smaller effects on output. When the MEI shock is sufficiently persistent, this expectations channel is quantitatively more important than the flattening of the Phillips Curve, and trend inflation exerts a dampening effect on the output response to a MEI shock.

To our knowledge, ours is the first paper to point out both the large interaction between trend inflation and the MEI shock and the dependence of that interaction on the persistence of the shock. There is a growing body of work suggesting that “investment shocks,” broadly defined, may play a key role in short run business cycle fluctuations. Fisher (2006) argues that IST shocks are an important source of fluctuations, with these shocks explaining half or more of the forecast error variances of output and hours at business cycle frequencies. Justiniano, Primiceri and Tambalotti (2010, 2011) argue that MEI shocks are perhaps the primary driver of business cycle fluctuations. Justiniano, Primiceri and Tambalotti (2011) argue that the MEI shock might proxy for a more fundamental shock to the functioning of the financial sector, which fits well with conventional wisdom concerning the cause and propagation of the recent Great Recession. Combining the empirical findings of the importance of investment shocks with our heretofore unreported results about the important interaction of MEI shocks with trend inflation, we argue that it is important from a positive perspective to take into account the effects of trend inflation in DSGE business cycle models.

6 Conclusion

Economists have recently debated the desirability for the Federal Reserve and other major central banks of the world to raise inflation targets. This debate is the result of economic pain experienced during and after the Great Recession. It would strain credulity to deny that a greater flexibility in lowering the nominal interest rate would have alleviated the burden of the last recession. Ireland (2011), for instance, argues that because of the zero lower bound on nominal interest rates, the Federal Reserve was prevented from stabilizing the U.S. economy as it previously did. With the
same flexibility the Federal Reserve had in the two previous recessions, the last one might have been shorter and less severe.

But proposals to raise the inflation target are built on the premise that it would not be costly to permanently increase trend inflation by a moderate amount, say from 2 to 4 percent at annualized rate. Surprisingly, despite the practical importance of this question, few efforts have been devoted prior to our study to address this question in the context of the empirically realistic medium-scale DSGE models that central bankers and academics use to study the macroeconomy. The evidence we have provided here offers a comprehensive benchmark against which these costs can be gauged in future research, and serves as a cautionary warning that the welfare costs of increasing an inflation target may be substantially higher than many think.

Another important side to our findings is their implications for the business cycle. For more than three decades now since the work of Kydland and Prescott (1982) and Nelson and Plosser (1982), macroeconomists have tried to identify the sources of business cycles. Moderate trend inflation can have strong distorting effects conditioned on shocks to the marginal efficiency of investment. These findings are thus complementary to those of Justiniano, Primiceri, and Tambalotti (2011), who show that MEI shocks are the most important driving force behind business cycles. The strong interaction between trend inflation and MEI shocks has been heretofore overlooked in the literature.

Given that much recent research ascribes an important role to MEI shocks, our analysis suggests that it is not innocuous to ignore trend inflation and that trend inflation may have larger effects on business cycle dynamics than previously thought. We are aware of no other paper which has explored the interaction between these shocks and trend inflation. Because there is an interesting dependence between the persistence of these shocks and the sign of the interaction between trend inflation and the responses of key aggregate variables to MEI shocks, we do not wish to take a stand on which direction trend inflation affects business cycle volatility. We simply note that there is a large interaction between trend inflation and these shocks, much larger than for either productivity or monetary shocks. Researchers ought to take this dependence into account when evaluating the quantitative effects and importance of these shocks.
References


Table 1: Non-Shock Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1/3</td>
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<td>$\eta$</td>
<td>6</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1</td>
</tr>
<tr>
<td>$b$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>3</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>5</td>
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<tr>
<td>$\psi_L$</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_K$</td>
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</tr>
</tbody>
</table>

Note: this table gives the baseline values of the parameters unrelated to the stochastic processes used in our quantitative simulations.

Table 2: Shock Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$g_A$</td>
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</tr>
<tr>
<td>$g_{c,t}$</td>
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<tr>
<td>$\rho_r$</td>
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</tr>
<tr>
<td>$s_r$</td>
<td>0.0020</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.81</td>
</tr>
<tr>
<td>$s_y$</td>
<td>0.0276</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.95</td>
</tr>
<tr>
<td>$s_A$</td>
<td>0.0030</td>
</tr>
</tbody>
</table>

Note: this table gives the baseline values of the parameters of the stochastic processes used in our quantitative simulations. The trend growth rate of the IST process is chosen to match the average growth rate of the relative price of investment goods in the data. The trend growth growth of the neutral productivity processes is chosen to match the average growth rate of output observed in the sample conditional on the growth rate of the IST process. Given the assumed values of autoregressive parameters governing the stochastic processes, the shock standard deviations are chosen to match the observed volatility of output growth in the data, with the MEI shock accounting for 50 percent of the variance of output growth, the neutral shock 35 percent, and the monetary shock 15 percent.

Table 3: Moments

<table>
<thead>
<tr>
<th>$E(\Delta Y)$</th>
<th>$\sigma(\Delta Y)$</th>
<th>$\sigma(\Delta I)$</th>
<th>$\sigma(\Delta C)$</th>
<th>$\sigma(\Delta L)$</th>
<th>$\sigma(Y^{hp})$</th>
<th>$\sigma(I^{hp})$</th>
<th>$\sigma(C^{hp})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.0057</td>
<td>0.0078</td>
<td>0.0251</td>
<td>0.040</td>
<td>0.0095</td>
<td>0.0166</td>
<td>0.0544</td>
</tr>
<tr>
<td>Data</td>
<td>(0.0057)</td>
<td>(0.0078)</td>
<td>(0.0202)</td>
<td>(0.0047)</td>
<td>(0.0079)</td>
<td>(0.0156)</td>
<td>(0.0302)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma(L^{hp})$</th>
<th>$\sigma(\pi)$</th>
<th>$\rho(\Delta Y, \Delta I)$</th>
<th>$\rho(\Delta Y, \Delta C)$</th>
<th>$\rho(\Delta Y, \Delta L)$</th>
<th>$\rho(Y^{hp}, I^{hp})$</th>
<th>$\rho(Y^{hp}, C^{hp})$</th>
<th>$\rho(Y^{hp}, L^{hp})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.0143</td>
<td>0.0044</td>
<td>0.9361</td>
<td>0.6314</td>
<td>0.3803</td>
<td>0.9317</td>
<td>0.5918</td>
</tr>
<tr>
<td>Data</td>
<td>(0.0171)</td>
<td>(0.0065)</td>
<td>(0.9192)</td>
<td>(0.7542)</td>
<td>(0.6313)</td>
<td>(0.9701)</td>
<td>(0.9053)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho_1(\Delta Y)$</th>
<th>$\rho_1(\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.6557</td>
</tr>
<tr>
<td>Data</td>
<td>(0.3634)</td>
</tr>
</tbody>
</table>

Note: this table shows selected moments generated from the baseline model. These moments are generated using the parameter values shown in the tables above with annualized trend inflation of 3.52 percent. “$\sigma$” denotes standard deviation, “$\Delta$” refers to the first difference operator, $\rho_1$ is a first order autocorrelation coefficient, and a superscript “$hp$” stands for the HP detrended component of a series using smoothing parameter of 1600. The variables $Y$, $I$, $C$, and $L$ are the natural logs of these series; $\pi$ is quarter-over-quarter inflation. Moments in the data are computed on the sample 1960q1-2007q3 and are shown in parentheses.
Table 4: Welfare Effects of Trend Inflation

<table>
<thead>
<tr>
<th>$\pi^*$</th>
<th>1.00 →</th>
<th>1.02 →</th>
<th>1.0352 →</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) Steady State</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0000</td>
<td>0</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>1.0200</td>
<td>0.0191</td>
<td>0</td>
<td>n/a</td>
</tr>
<tr>
<td>1.0352</td>
<td>0.0449</td>
<td>0.0263</td>
<td>0</td>
</tr>
<tr>
<td>1.0400</td>
<td>0.0557</td>
<td>0.0373</td>
<td>0.0113</td>
</tr>
<tr>
<td>(a) Means</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0000</td>
<td>0</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>1.0200</td>
<td>0.0222</td>
<td>0</td>
<td>n/a</td>
</tr>
<tr>
<td>1.0352</td>
<td>0.0519</td>
<td>0.0303</td>
<td>0</td>
</tr>
<tr>
<td>1.0400</td>
<td>0.0643</td>
<td>0.0430</td>
<td>0.0131</td>
</tr>
</tbody>
</table>

Note: this table shows consumption equivalent welfare losses from increasing the trend inflation rate using the benchmark parameterization of our model. Panels marked (a) show losses based on the non-stochastic steady state, while panels marked (b) present losses based on stochastic means. A number in the table has the interpretation as the fraction of consumption the representative household would be willing to give up to avoid changing the trend inflation rate from the level in the columns to the level shown in the rows.

Table 5: Welfare Effects of Trend Inflation, Alternative Model Specifications

<table>
<thead>
<tr>
<th>$\pi^*$: 1.02 → 1.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>(i) Flexible wages</td>
</tr>
<tr>
<td>(ii) No trend growth</td>
</tr>
<tr>
<td>(iii) All growth from neutral productivity</td>
</tr>
<tr>
<td>(iv) All growth from IST</td>
</tr>
<tr>
<td>(v) Flexible wages, no growth</td>
</tr>
<tr>
<td>(vi) Flexible prices</td>
</tr>
</tbody>
</table>

Note: this table presents consumption equivalent welfare losses from higher trend inflation under four different deviations from the benchmark model: flexible wages, $\xi_w = 0$, in panel (i); no trend growth in panel (ii); no roundabout production, $\phi = 0$, in panel (ii); and no trend growth, flexible prices, and no roundabout production in panel (iv). The rows and columns are organized in the same manner as Table 4.

Table 6: Welfare Effects of Trend Inflation, Alternative Model Specifications

<table>
<thead>
<tr>
<th>$\pi^*$: 1.02 → 1.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>(i) No RP</td>
</tr>
<tr>
<td>(ii) No EB</td>
</tr>
<tr>
<td>(iii) Only wage subject to borrowing</td>
</tr>
<tr>
<td>(iv) No MEI shocks</td>
</tr>
<tr>
<td>(v) No RP, EB, or MEI shocks</td>
</tr>
<tr>
<td>(vi) No RP, EB, or MEI shocks, flexible wages, no growth</td>
</tr>
<tr>
<td>(vii) No productivity shocks</td>
</tr>
<tr>
<td>(viii) More persistent MEI shock ($\rho_I = 0.95$)</td>
</tr>
<tr>
<td>(ix) Less persistent MEI shock ($\rho_I = 0.66$)</td>
</tr>
</tbody>
</table>

Note: this table presents consumption equivalent welfare losses from higher trend inflation under several different deviations from the baseline model. These deviations are described in the left column. The rows and columns are organized in the same manner as Table 4.
Table 7: Welfare Effects of Trend Inflation, Parameter Robustness

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Steady State</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Infinite Frisch elasticity ($\chi = 0$)</td>
<td>0.0145</td>
<td>0.0149</td>
</tr>
<tr>
<td>(ii) Full wage indexation</td>
<td>0.0093</td>
<td>0.0099</td>
</tr>
<tr>
<td>(iii) Partial wage indexation (0.5)</td>
<td>0.0174</td>
<td>0.0190</td>
</tr>
<tr>
<td>(iv) Partial wage indexation (0.25)</td>
<td>0.0254</td>
<td>0.0285</td>
</tr>
<tr>
<td>(v) Full price indexation</td>
<td>0.0332</td>
<td>0.0377</td>
</tr>
</tbody>
</table>

Note: this table presents consumption equivalent welfare losses from higher trend inflation under several different deviations from the baseline model. These deviations are described in the left column. The rows and columns are organized in the same manner as Table 4.
Note: this figure plots impulse responses of output and inflation to the three shocks in our model – productivity, marginal efficiency of investment (MEI), and the monetary policy shock – for three different annualized rates of trend inflation.
Figure 2: Trend Inflation and the Consumption Response to a MEI Shock

Note: this figure plots the impulse response of consumption to the MEI shock for three different annualized rates of trend inflation.

Figure 3: Model Features and the Response of Consumption to MEI Shock

Note: this figure plots the impulse response of consumption to the MEI shock for three different annualized rates of trend inflation. We do so for four different cases with different features of our model turned off.
Figure 4: Trend Inflation and the Persistence of the MEI Shock

Note: this figure plots impulse responses of output to the MEI shock for three different levels of trend inflation. The different panels consider different values of the persistence parameter for the MEI shock, $\rho_I$. 
Figure 5: Interaction Between Trend Inflation and the Persistence of the MEI Shock

Note: this figure plots the absolute difference between the impulse response of output at a ten quarter horizon with four percent trend inflation and zero percent trend inflation against different values of $\rho_I$. 
A Full Set of Stationarized Equilibrium Conditions

This Appendix lists the full set of stationarized equations which characterize the equilibrium of our model. Variables with a $\sim$ denote transformed variables which are stationary. After listing the equilibrium conditions, we discuss our measure of welfare and construction of consumption equivalent welfare losses.

$$\bar{X}_t = \frac{1}{C_t - bg_{\gamma}^{-1}C_{t-1}} - E_t \frac{\beta b}{g_{\gamma} C_{t+1} - bC_t} \quad (A1)$$

$$\bar{r}_t = \gamma_1 + \gamma_2 (Z_t - 1) \quad (A2)$$

$$\tilde{X}_t = \bar{\mu}_t \frac{1 - \frac{k}{2} \left( \frac{\bar{I}_t}{\bar{I}_{t-1}} \right) g_{\gamma} - g_{\gamma}^T}{\bar{I}_{t-1}} - \kappa \left( \frac{\bar{I}_t}{\bar{I}_{t-1}} g_{\gamma} - g_{\gamma}^T \right) + \beta E_t g_{\gamma}^{-1} \bar{\mu}_{t+1} \delta_{t+1} \kappa \left( \frac{\bar{I}_{t+1}}{\bar{I}_t} g_{\gamma} - g_{\gamma}^T \right) \left( \frac{\bar{I}_{t+1}}{\bar{I}_t} g_{\gamma} \right) \quad (A3)$$

$$g_t g_{\gamma} \bar{\mu}_t = \beta E_t \tilde{X}_{t+1} \left( \frac{g_{\gamma}^k T_{t+1}}{T_{t+1}} - \left( \gamma_1 (Z_{t+1} - 1) + \frac{\gamma_2}{2} (Z_{t+1} - 1)^2 \right) \right) + \beta (1 - \delta) E_t \bar{\mu}_{t+1} \quad (A4)$$

$$\tilde{X}_t = \beta g_{\gamma}^{-1} E_t (1 + i_t) \pi_{t+1}^{-1} \tilde{X}_{t+1} \quad (A5)$$

$$\tilde{w}_t^* = \frac{\sigma}{\sigma - 1} \tilde{f}_{1,t} \quad (A6)$$

$$\tilde{f}_{1,t} = \eta \left( \frac{\tilde{w}_t}{\tilde{w}_t^*} \right)^{\sigma(1 + \chi)} L_t^{1 + \chi} + \beta \xi_w E_t (\pi_{t+1})^{\sigma(1 + \chi)} \left( \frac{\tilde{w}_{t+1}}{\tilde{w}_t^*} \right)^{\sigma(1 + \chi)} g_{\gamma} \tilde{f}_{1,t+1} \quad (A7)$$

$$\tilde{f}_{2,t} = \tilde{\lambda}_t \left( \frac{\tilde{w}_t}{\tilde{w}_t^*} \right) \sigma L_t + \beta \xi_w E_t (\pi_{t+1})^{\sigma - 1} \left( \frac{\tilde{w}_{t+1}}{\tilde{w}_t^*} \right) \sigma^{-1} \tilde{f}_{2,t+1} \quad (A8)$$

$$\tilde{K}_t = g_t g_{\gamma} \alpha (1 - \phi) \frac{mc_t}{\tilde{r}_t^k} \left( s_t \tilde{X}_t + F \right) \quad (A9)$$

$$L_t = (1 - \alpha) \left( 1 - \phi \right) \frac{mc_t}{\tilde{w}_t^*} \left( s_t \tilde{X}_t + F \right) \quad (A10)$$

$$\tilde{\Gamma}_t = \phi mc_t \left( s_t \tilde{X}_t + F \right) \quad (A11)$$

$$p_t^* = \frac{\theta}{\theta - \frac{1}{x_t^1}} x_t^1 \quad (A12)$$

$$x_t^1 = \tilde{X}_t mc_t \tilde{X}_t + \xi_p \beta \left( \frac{1}{\pi_{t+1}} \right)^{-\theta} x_{t+1}^1 \quad (A13)$$

$$x_t^2 = \tilde{X}_t \tilde{X}_t + \xi_p \beta \left( \frac{1}{\pi_{t+1}} \right)^{1 - \theta} x_{t+1}^2 \quad (A14)$$

$$1 = \xi_p \left( \frac{1}{\pi_t} \right)^{1 - \theta} + (1 - \xi_p) p_t^{1 - \theta} \quad (A15)$$

$$\tilde{w}_t^{1 - \sigma} = \xi_w g_{\gamma}^{\sigma - 1} \left( \frac{\tilde{w}_{t-1}}{\tilde{w}_t^*} \right)^{1 - \sigma} + (1 - \xi_w) \tilde{w}_t^{1 - \sigma} \quad (A16)$$

$$\tilde{Y}_t = \tilde{X}_t - \tilde{\Gamma}_t \quad (A17)$$
\[ s_t \tilde{X}_t = A_t \tilde{Y}_t \sim (1-\phi) L_t^{(1-\phi)} g_Y^{\alpha(\phi-1)} g_I^{\alpha(\phi-1)} - F \quad (A18) \]

\[ \bar{Y}_t = \bar{C}_t + \bar{I}_t + g^{-1}_t g^{-1}_t \left( \gamma_1 (Z_t - 1) + \frac{\gamma_2}{2} (Z_t - 1)^2 \right) \bar{K}_t \quad (A19) \]

\[ \bar{K}_{t+1} = \partial_t \left( 1 - \frac{\kappa}{2} \left( \frac{\bar{I}_t}{1-\delta} g_T - g_T \right) \right) \bar{I}_t + (1 - \delta) g^{-1}_T g^{-1}_T \bar{K}_t \quad (A20) \]

\[ \frac{1 + i_t}{1 + \bar{I}_t} = \left( \frac{\pi_t}{\pi} \right)^{\alpha_g} \left( \frac{\bar{Y}_t}{Y_{t-1}} \right)^{1-\rho_g} \left( \frac{1 + i_{t-1}}{1 + \bar{I}_t} \right)^{\rho_i} \varepsilon_t \quad (A21) \]

\[ \bar{K}_t = Z_t \tilde{K}_t \quad (A22) \]

\[ s_t = (1 - \xi_p) p_t^{\alpha} + \xi_p \left( \frac{1}{\pi_t} \right)^{-\theta} s_{t-1} \quad (A23) \]

\[ v_t^{w} = (1 - \xi_w) \left( \frac{\bar{w}_t^{w}}{\tilde{w}_t^{w}} \right)^{-(1+\chi)} + \xi_w \left( \frac{\bar{w}_t^{w}}{\tilde{w}_t^{w}} g_T^{-1} \frac{1}{\pi_t} \right)^{-(1+\chi)} v_{t-1}^{w} \quad (A24) \]

\[ \bar{V}_t^c = \ln \left( \bar{C}_t - b g_T^{-1} \bar{C}_{t-1} \right) + \beta E_t \bar{V}_{t+1}^c \quad (A25) \]

\[ V_t^n = -\eta_t \frac{L_t^{1+\chi}}{1+\chi} v_t^{w} + \beta E_t V_{t+1}^n \quad (A26) \]

\[ W_t = \bar{V}_t^c + \bar{V}_t^n + \Psi_t \quad (A27) \]

\[ \Psi_t = \frac{\beta \ln g_T}{(1-\beta)^2} \quad (A28) \]

\[ \bar{A}_t = (\bar{A}_{t-1})^{\nu} \exp \left( s_A u_t^A \right) \quad (A29) \]

\[ \partial_t = (\partial_{t-1})^{\nu} \exp \left( s_f u_t^f \right) \quad (A30) \]

In these equations \( g_T = \frac{\bar{X}_t}{\bar{I}_t} \), or the growth rate of the deterministic trend. We require that \( \beta g_T < 1 \). The recursive representation of social welfare above is written as the sum of three components: utility from consumption, labor, and a third term. The third term, defined as \( \Psi_t \) in (A28), is essentially a shift term that arises because of trend growth and appears when rewriting flow utility from consumption in terms of stationaryized consumption. Recursive utility from consumption and labor in the levels, are, respectively:

\[ V_t^c = \ln \left( C_t - b C_{t-1} \right) + \beta E_t V_{t+1}^c \quad (A31) \]

\[ V_t^L = -\eta_t \frac{L_t^{1+\chi}}{1+\chi} v_t^{w} + \beta E_t V_{t+1}^L \quad (A32) \]

Writing the recursive representation for utility from consumption in terms of stationary variables, one has:

\[ V_t^c = \ln \left( \bar{C}_t - b g_T^{-1} \bar{C}_{t-1} \right) + \ln \bar{Y}_t + \beta E_t V_{t+1}^c \quad (A33) \]
Define:

\[ \tilde{V}_t^c = \ln \left( \tilde{C}_t - bg^{-1} \tilde{C}_{t-1} \right) + \beta E_t \tilde{V}_{t+1}^c \]  
(A34)

Similarly, define \( \Psi_t \) as:

\[ \Psi_t = E_t \sum_{s=0}^{\infty} \beta^s E_t \ln \Upsilon_{t+s} \]  
(A35)

If we normalize \( \Upsilon_t = 1 \), then \( E_t \ln \Upsilon_{t+s} = s g_{\Upsilon} \). Then we can write:

\[ \Psi_t = \frac{\beta \ln g_{\Upsilon}}{(1 - \beta)^2} \]  
(A36)

Then aggregate welfare can be written as:

\[ V_t = \tilde{V}_t^c + \Psi_t + V_t^L \]  
(A37)

We define the consumption equivalent, \( \lambda \), as the fraction of consumption that would have to be sacrificed in each period in a benchmark case (e.g. two percent inflation) to have the same welfare as in an alternative case (e.g. four percent inflation). As discussed in the text, we consider two different consumption equivalents – one based on steady states, \( \lambda_{ss} \), and one based on stochastic means. Given the definition of welfare and the fact that utility over consumption is logarithmic, it is straightforward to derive either measure, which are both shown below.

\[ \lambda_{ss} = 1 - \exp \left[ (1 - \beta)(V_{SS} - V_{SS}^B) \right] \]  
(A38)

\[ \lambda_m = 1 - \exp \left[ (1 - \beta)(E(V) - E(V_B)) \right] \]  
(A39)

In the above expressions a subscript “B” denotes the “base” scenario (e.g. two percent inflation) while the absence of a subscript denotes the alternative scenario (e.g. four percent inflation). A superscript “SS” stands for the non-stochastic steady state, while \( E(\cdot) \) is the unconditional expectations operator.