This document details several stylized facts about post-war US macroeconomic data. When evaluating dynamic economic models, it is always important to keep in mind the stylized facts we want those models to be able to reproduce. The data used in this document are publicly available. The NIPA data were downloaded from the St. Louis Fed FRED website. The employment and population data are from unpublished BLS data available on Valerie Ramey’s website. The NIPA data are available at a quarterly frequency beginning in 1947q1 and continuing through 2009. The employment and population data begin in 1948 and end in 2007. All data are seasonally adjusted and are expressed as natural logs.

A starting point is to note that there has been substantial population growth. Growth in economic aggregates due to population growth is beyond the purview of modern macroeconomics. Below is a plot of the (log) of the adult population across time:

We can see that population growth is very smooth (although this is partly due to the fact that the quarterly numbers are interpolated from an annual series). We can observe a bit of an uptick in the adult population from the 1960s to about 1980 (this is the effect of the baby boomers coming of age) and a subsequent slowdown thereafter. The average growth rate of population (i.e. the average first difference of the log) is 0.0033, or about a third of a percent. This translates into an average annual growth rate of about 1.25 percent (ignoring the (small) effects of compounding). Since we’re not interested in studying population growth, we want to remove this from most of our series. As such, except where indicated, I will be talking about per capita series (i.e. log GDP - log Pop . . . noting that the units/scales have little meaning, so negative numbers are perfectly fine).

Below is a plot of three main aggregates of interest – GDP (i.e. $Y$), non-durables + services consumption (i.e. $C$), and investment plus purchases of durable goods (i.e. $I$). Although they are accounted for separately by NIPA, it is common to group durables with investment. All series are logged and in per capita terms:
A couple of features stand out. First, all three series grow over time at what appears to be roughly the same rate. As a corollary, the ratios of these series appear to be roughly constant. Third, investment appears much more volatile than the other two series, while consumption appears to be the least volatile. Because I've already taken out population growth, this growth cannot be solely due to population growth. The average growth rate of both GDP and non-durables and services consumption is 0.005, or about 2.00 percent at annual rate. Mean investment growth is somewhat larger (you can see this in the plot of the series) at 0.007, or about 2.8 percent per year.

Next consider time series plots of different measures of employment. Below is a plot of total hours worked and total employment, both in logs and both expressed in per capita terms.

Total hours worked (in blue, on left scale) moves around a fair amount, but it appears to be roughly mean-reverting. There is an upward trend in total employment per capita, which basically just reflects an increase in labor force participation, particularly among females. It's clear that these two series move very much together. Given that total hours worked
appears to be roughly mean-reverting while there is an upward trend in total employment, it
must be the case that average hours worked (i.e. total hours divided by total employment)
must be trending down. We in fact see this. People today work, on average, about 10
percent less than they did 50 years ago (i.e. from a 40 to 36 hour workweek).

The aggregate price level shows a strong upward trend. Below is a plot of the log GDP
implicit price deflator (this series is not in per capita terms):

![GDP Deflator](image)

Average inflation over the time period (defined as the log first difference of the price level)
is 0.008, which comes out to a little over 3 percent at an annualized rate. We can tell from
the picture that inflation was higher from about 1970 to the early 1980s (corresponding with
the oil “shocks”), and lower both before and after.

Given that output is clearly trending up over time while total hours appear loosely
stationary, it must be the case the average labor productivity is rising over time. Below is
a plot of labor productivity (defined as the log difference between GDP and total hours):
Labor productivity seems fairly volatile, but also displays a clear upward trend. If you look hard enough, you can see the “productivity slowdown” in the 1970s. Average growth in labor productivity over the post-war period has been about 0.005, or about 2 percent at an annualized rate. Not surprisingly, this is basically the same as the average growth rate of GDP itself.

We want a way to separate out the obvious trends in these data from the cycle. The definition of a “cycle” is not particularly well-defined. Most definitions pin down cycles as movements in economic activity with periodicities between six and thirty-two quarters (i.e. it takes the variable between six and thirty-two quarters to go through a complete “cycle” from peak to trough). Perhaps the most popular way of isolating the cyclical component of a series is by using the HP filter. For quarterly data, it is common to use a smoothing parameter of 1600. A smoothing parameter of infinity gives rise to a linear trend and the cycle being defined as deviations about that linear trend.

Below is a plot of the HP detrended component of output. The shaded areas represent recessions as defined by the NBER:

Contrary to popular perception, there is no formal definition of a recession (it is commonly thought to be two consecutive quarters of decline in real GDP). Rather, the NBER dates
recessions (after the fact, of course) looking at a variety of economic indicators, including employment, industrial production, etc. Not surprisingly, however, the NBER recession dates line up quite well with the troughs in activity picked out by the HP filter.

Below are time series plots of the cyclical component of output along with the cyclical component of other series:
There are a couple of things to be gleaned from these pictures. First, consumption, investment, hours, average hours, employment, and labor productivity all appear to be procyclical in the sense that their cyclical component appears to be positively correlated with the cyclical component of output. The price level appears relatively acyclical, as there are episodes where the correlation looks negative (early 80s recessions) and other episodes where it looks positive (late 40s recession). Secondly, consumption appears much less volatile than output, while investment appears much more volatile than output. Third, total hours and total employment are both about as volatile as output, while average hours worked is much less volatile than output. Fourth, labor productivity is procyclical, but appears to be less volatile than output. Fifth, it is clear that most of these series are fairly persistent in the sense that a positive realization tends to be followed by more positive realizations and so on.

The table below formalizes some of these observations. It shows the volatility of each series (standard deviation), the relative volatility with output (ratio of standard deviations), the autocorrelation coefficient, and the contemporaneous correlation with output.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Volatility</th>
<th>Relative Volatility</th>
<th>Autocorrelation</th>
<th>Correlation with Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.0165</td>
<td>1.0000</td>
<td>0.8390</td>
<td>1.0000</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0079</td>
<td>0.4788</td>
<td>0.8390</td>
<td>0.7794</td>
</tr>
<tr>
<td>Investment</td>
<td>0.0666</td>
<td>4.0364</td>
<td>0.8104</td>
<td>0.8655</td>
</tr>
<tr>
<td>Total Hours</td>
<td>0.0150</td>
<td>0.9091</td>
<td>0.8829</td>
<td>0.8650</td>
</tr>
<tr>
<td>Total Employment</td>
<td>0.0122</td>
<td>0.7394</td>
<td>0.9064</td>
<td>0.8004</td>
</tr>
<tr>
<td>Average Hours</td>
<td>0.0045</td>
<td>0.2727</td>
<td>0.6880</td>
<td>0.7054</td>
</tr>
<tr>
<td>Price Level</td>
<td>0.0094</td>
<td>0.5697</td>
<td>0.9049</td>
<td>-0.2207</td>
</tr>
<tr>
<td>Labor Productivity</td>
<td>0.0083</td>
<td>0.5030</td>
<td>0.6470</td>
<td>0.4208</td>
</tr>
</tbody>
</table>

This table confirms many of our casual observations from just eyeballing the data. Consumption is less volatile than output, while investment is much more volatile than output. Hours and employment are almost as volatile as output, while average hours are not nearly as volatile as employment. This finding suggests that it is the extensive margin (hiring and
firing workers) that is much more important over the business cycle than the intensive margin (increasing or reducing the hours of existing employees). This fact turns out to be one of the few major sticking points of real business cycle theory, as it typically has no extensive margin. The price level is mildly countercyclical. Labor productivity is procyclical and about half as volatile as output. All of the series are fairly persistent in the sense of having autocorrelation coefficients close to one.

Real business cycle theory in particular emphasizes the role of productivity shocks in generating fluctuations. Labor productivity could move around in response to changes in employment due to “demand shocks” and so forth, and is thus an imperfect measure of the true productive capability of the economy. A different (and maybe better) measure of productivity is total factor productivity, or the Solow residual. Calculations of TFP typically assume a constant returns to scale Cobb-Douglas production function:

$$y_t = a_t k_t^\alpha l_t^{1-\alpha}$$

The parameter $\alpha$ can be measured as one minus labor’s share of total income. In US data it is fairly stable at round 0.33. We already have a good measure of labor input (total hours), and a measure of total output. If we had a measure of the physical capital stock, we could determine TFP as a the residual factor (i.e. the production we cannot explain with measured inputs). In particular, in growth rate form (i.e. take logs and then first difference):

$$g_{at} = g_{yt} - \alpha g_{kt} - (1 - \alpha)g_{nt}$$

The difficulty here is that we typically do not have good measures of the capital stock at quarterly frequencies. We can get measures of the capital stock at annual frequencies. In particular, the BLS capital services measure is the capital stock at the end of the year. The question is then how to construct a quarterly measure of the capital stock. The most common (and easy) way to proceed is by using the perpetual inventory method. Given an initial value for the capital stock, the quarterly version can be constructed using data on quarterly investment. Take the standard accumulation equation:

$$k_t = i_{t-1} + (1 - \delta)k_{t-1}$$

If you solve this backwards, you get:

$$k_t = (1 - \delta)^t k_0 + \sum_{j=0}^{t-1} i_{t-j} (1 - \delta)^j$$

All we need to proceed is quarterly data on investment, a measure of depreciation, and an initial condition for the capital stock. Because we have annual capital stock data, we can use one of the end of year estimates of the capital stock as our initial condition (in practice I use 1948 as the initial condition). Then I can create a series using investment (note for this exercise I do not include consumer durables in the measure of investment). I use a fixed value of $\delta = 0.03$, which implies an annual depreciation rate of a little over 10 percent. In reality, there are more sophisticated tools to come up with time-varying depreciation rates, and there are reasons to think that depreciation rates have trended up over time (we’ve transitioned from mainly structures, which have low depreciation, to things like computers,
which depreciate quickly). Below is a plot of the resulting quarterly series, as well as a plot of the HP detrended capital stock along with HP detrended GDP. The capital stock is substantially smoother than GDP and lags GDP a bit. That capital lags GDP is not surprising given that investment is strongly correlated with GDP contemporaneously:

Given the measure of the capital stock and measures of GDP and labor input (total hours), I can proceed as above in calculating a TFP series. The series as I construct it is in growth rates. I cumulate the growth rates to create the corresponding series in log levels. Below are graphs of the level of log TFP and the cyclical component of TFP along with the cyclical component of output.

It is quite evident from the picture that TFP is procyclical and a little less volatile than GDP. This conjecture is borne out in the data. The relative volatility of TFP is about 0.62 (standard deviation of 0.0103 vs. 0.0165. The contemporaneous correlation of TFP with detrended output is 0.78, and the first order autocorrelation of TFP is 0.75.

I should note that there is large debate about how well TFP measures “true” technology. One sticking point in particular is that factor utilization is procyclical. Factor utilization refers to the intensity with which factors (capital and labor) are used. Put differently, when times are good, factories work their capital and workers harder than when business is slow. Because our measures of factors (capital, in particular) have no way of capturing this, the
cyclicality of TFP may be biased upwards. For this reason, some authors have come up with ways to “correct" TFP for variable utilization (e.g. Basu, Fernald, and Kimball (2006)). This is beyond the realm of this course.