New Keynesian Model with Price and Wage Stickiness

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June 19, 2020

This note describes a basic New Keynesian model (no capital and no other real frictions) where both prices and wages are sticky. Rather than doing the setup of Erceg, Henderson, and Levin (2000, JME), I do a union setup where I can assume a representative household.

1 Household

Household flow utility is given by:

$$U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi}$$

Flow utility is discounted by $\beta$. The budget constraint facing the household, written in nominal terms, is:

$$P_t C_t + B_t \leq MRS_t L_t + R_{t-1} B_{t-1} + DIV_t$$

The household can save via a one period bond with gross nominal interest rate $R_t$. $MRS_t$ is the nominal remuneration for supply labor to unions. A Lagrangian is:

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} + \mu_t [MRS_t L_t + R_{t-1} B_{t-1} + DIV_t - P_t C_t - B_t] \right\}$$

The first order conditions are:

$$C_t^{-\sigma} = \mu_t P_t$$
$$\psi L_t^\chi = \mu_t MRS_t$$
$$\mu_t = \beta R_t \mathbb{E}_t \mu_{t+1}$$

Re-written in real terms, where $\Pi_t = P_t/P_{t-1}$, we have:

$$\psi L_t^\chi = C_t^{-\sigma} mrs_t$$

$$1 = R_t \mathbb{E}_t A_{t,t+1} \Pi_{t+1}^{-1}$$
\[ \Lambda_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \]  

(3)

\( mrs_t = MRS_t / P_t \) is the real remuneration for supply labor. \( \Lambda_{t,t+1} \) is the real stochastic discount factor.

2 Labor Markets

There are a continuum of labor unions indexed by \( l \in [0, 1] \). They hire labor from the household at \( MRS_t \) and sell to a labor packer at \( W_t(h) \). The labor packer combines union labor into a final labor input available to firms via a CES technology. In particular:

\[ L_{d,t} = \left[ \int_0^1 L_t(l) \frac{\epsilon_w - 1}{\epsilon_w} dl \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \]

Profit maximization yields a demand curve for each union’s labor and an aggregate wage index:

\[ L_t(l) = \left( \frac{W_t(l)}{W_t} \right)^{-\epsilon_w} L_{d,t} \]

\[ W_t^{1-\epsilon_w} = \int_0^1 W_t(l)^{1-\epsilon_w} dl \]

Unions simply repackage labor from the household one-for-one for resale to the packer. Nominal dividends are:

\[ DIV_{u,t}(l) = W_t(l) L_t(l) - MRS_t L_t(l) \]

Plugging in the demand function:

\[ DIV_{u,t}(l) = W_t(l)^{1-\epsilon_w} W_t^{\epsilon_w} L_{d,t} - MRS_t W_t(l)^{-\epsilon_w} W_t^{\epsilon_w} L_{d,t} \]

Dividing by \( P_t \) to put this into real terms:

\[ div_{u,t}(l) = W_t(l)^{1-\epsilon_w} W_t^{\epsilon_w} P_t^{-1} L_{d,t} - mrs_t W_t(l)^{-\epsilon_w} W_t^{\epsilon_w} L_{d,t} \]

With probability 1 - \( \phi_w \), a union can update its wage. The problem for a union given the opportunity to update is to pick \( W_t(l) \) to maximize the present discounted value of real dividends, where discounting is by the household’s SDF as well as the probability that a price chosen today will be in effect in the future. The problem is:

\[ \max_{W_t(l)} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} \left\{ W_t(l)^{1-\epsilon_w} W_t^{\epsilon_w} P_t^{-1} L_{d,t} - mrs_t W_t(l)^{-\epsilon_w} W_t^{\epsilon_w} L_{d,t} \right\} \]

The FOC is:
\[(1-\epsilon_w)W_t(l)^{-\epsilon_w} \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} W_t^{\epsilon_w} L_{t,t+j} + \epsilon_w W_t(l)^{-\epsilon_w-1} \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} m r s_{t+j} W_t^{\epsilon_w} L_{t,t+j} = 0\]

The reset wage doesn't depend upon \(l\) indexes, so I will call the optimal reset wage \(W_t^\#\). The FOC can be written:

\[
W_t^\# = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} m r s_{t+j} W_t^{\epsilon_w} L_{t,t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} W_t^{\epsilon_w} L_{t,t+j}}
\]

This can be written recursively:

\[
W_t^\# = \frac{\epsilon_w}{\epsilon_w - 1} \frac{F_{1,t}}{F_{2,t}}
\]

\[
F_{1,t} = m r s_t W_t^{\epsilon_w} L_{t,d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} F_{1,t+1}
\]

\[
F_{2,t} = W_t^{\epsilon_w} P_t^{-1} L_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} F_{2,t+1}
\]

Write \(F_{1,t}\) and \(F_{2,t}\) in terms of real variables by multiplying and dividing by powers of \(P_t\):

\[
F_{1,t} = m r s_t w_t^{\epsilon_w} P_t^{\epsilon_w} L_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} F_{1,t+1}
\]

\[
F_{2,t} = w_t^{\epsilon_w} P_t^{-1} L_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} F_{2,t+1}
\]

Define \(f_{1,t} = F_{1,t}/P_t^{\epsilon_w}\) and \(f_{2,t} = F_{2,t}/P_t^{\epsilon_w-1}\). We then have:

\[
w_t^\# = \frac{\epsilon_w}{\epsilon_w - 1} \frac{f_{1,t}}{f_{2,t}}
\]

\[
f_{1,t} = m r s_t w_t^{\epsilon_w} L_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_w} f_{1,t+1}
\]

\[
f_{2,t} = w_t^{\epsilon_w} L_{d,t} + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_w-1} f_{2,t+1}
\]

### 3 Production

Production is split into three sectors. A representative wholesale firm hires labor from the labor packer and produces output, selling it to a continuum of retail firms at \(P_w,t\). The retail firms purchase wholesale output at \(P_w,t\), costly repackage it, and sell it to a competitive final goods firm at \(P_t(f)\), where retailers are indexed by \(f \in [0, 1]\). The final goods firm combines retail output into a final output good.

Retail output is transformed into final output via:
\[ Y_t = \left[ \int_0^1 Y_t(f)^{-\epsilon_p} \frac{\epsilon_p - 1}{\epsilon_p} df \right]^{\epsilon_p} \]

Profit maximization by the final goods firm yields a demand for each retail output and a price index.

\[ Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\epsilon_p} Y_t \]

\[ P_t^{1-\epsilon_p} = \int_0^1 P_t(f)^{1-\epsilon_p} df \]

Retailers costlessly transform wholesale output into retail output. Their nominal dividend is:

\[ DIV_{r,t}(f) = P_t(f)Y_t(f) - P_{w,t}Y_t(f) \]

Using the demand function, this is:

\[ D_{r,t}(f) = P_t(f)^{1-\epsilon_p} P_t^{\epsilon_p} Y_t - P_{w,t} P_t(f)^{-\epsilon_p} P_t^{\epsilon_p} Y_t \]

Or, in real terms:

\[ d_{r,t}(f) = P_t(f)^{1-\epsilon_p} P_t^{\epsilon_p-1} Y_t - P_{w,t} P_t(f)^{-\epsilon_p} P_t^{\epsilon_p-1} Y_t \]

Retailers can only adjust their price with probability 1 – \( \phi_p \). This makes their price-setting problem dynamic, where future real dividends are discounted by the household’s stochastic discount factor as well as the probability that a price chosen in period \( t \) remains in effect in the future. The price-setting problem is:

\[
\max_{P_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} \phi^j_p \Lambda_{t,t+j} \left\{ P_t(f)^{1-\epsilon_p} P_t^{\epsilon_p-1} Y_{t+j} - P_{w,t} P_t(f)^{-\epsilon_p} P_t^{\epsilon_p-1} Y_{t+j} \right\}
\]

The first order condition is:

\[
(1 - \epsilon_p)P_t(f)^{-\epsilon_p} \mathbb{E}_t \sum_{j=0}^{\infty} \phi^j_p \Lambda_{t,t+j} P_t^{\epsilon_p-1} Y_{t+j} + \epsilon_p P_t(f)^{-\epsilon_p-1} \mathbb{E}_t \sum_{j=0}^{\infty} \phi^j_p \Lambda_{t,t+j} P_{w,t} P_t^{\epsilon_p-1} Y_{t+j} = 0
\]

The optimal reset price does not depend on \( f \). Call it \( P_t^\# \). We can re-write the FOC as:

\[
P_t^\# = \frac{\epsilon_p}{\epsilon_p - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \phi^j_p \Lambda_{t,t+j} P_{w,t} P_t^{\epsilon_p-1} Y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \phi^j_p \Lambda_{t,t+j} P_t^{\epsilon_p-1} Y_{t+j}}
\]
This can be written recursively:

\[ P_t^# = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1,t}}{X_{2,t}} \]

\[ X_{1,t} = p_w,t P_{t}^{\epsilon_p} Y_t + \phi_p A_{t,t+1} X_{1,t+1} \]

\[ X_{2,t} = P_{t}^{\epsilon_p-1} Y_t + \phi_p A_{t,t+1} X_{2,t+1} \]

Where \( p_w,t = P_{w,t}/P_t \) and is interpretable as real marginal cost. Define \( x_{1,t} = X_{1,t}/P_{t}^{\epsilon_p} \) and \( x_{2,t} = X_{2,t}/P_{t}^{\epsilon_p-1} \). We have:

\[ x_{1,t} = p_w,t Y_t + \phi_p E_t A_{t,t+1} \Pi_{t+1}^{\epsilon_p} x_{1,t+1} \]  

\[ x_{2,t} = Y_t + \phi_p E_t A_{t,t+1} \Pi_{t+1}^{\epsilon_p-1} x_{2,t+1} \]

\[ \Pi_{t}^# = \frac{\epsilon_p}{\epsilon_p - 1} \frac{x_{1,t}}{x_{2,t}} \]  

Where \( \Pi_t = P_t/P_{t-1} \) and \( \Pi_{t}^# = P_{t}^#/P_t \).

The wholesale firm produces output according to:

\[ Y_{W,t} = A_t L_{d,t} \]  

Its nominal dividend is:

\[ DV_{W,t} = P_{w,t} Y_{W,t} - W_t L_{d,t} \]

The optimality condition is:

\[ W_t = P_{w,t} A_t \]

Or, in real terms:

\[ w_t = p_{w,t} A_t \]

4 Monetary Policy

Assuming the gross nominal rate, \( R_t \), is set according to a Taylor type rule:

\[ \ln R_t = (1 - \rho R) \ln R + \rho R \ln R_{t-1} + (1 - \rho R) \theta \pi (\ln \Pi_t - \ln \Pi) + s R \varepsilon R,t \]  

Variables without time subscripts denote non-stochastic steady state values.
5 Aggregation

The aggregate inflation rate and real wage evolve according to the following expressions, which can be derived using properties of Calvo pricing:

\[ 1 = (1 - \phi_p) \left( \Pi_t^p \right)^{1-\epsilon_p} + \phi_p \Pi_t^{\epsilon_p} - 1 \] (13)

\[ w_t^{1-\epsilon_w} = (1 - \phi_w) \left( w_t^w \right)^{1-\epsilon_w} + \phi_w \Pi_t^{\epsilon_w} - 1 w_{t-1}^{1-\epsilon_w} \] (14)

Goods market-clearing requires that wholesale output by sold to unions in the aggregate, or:

\[ Y_{W,t} = \int_0^1 Y_t(f) df \]

Given the demand function for each retailer's output, this works out to:

\[ Y_{W,t} = Y_t v_t^p \] (15)

Where \( v_t^p \) is a measure of price dispersion:

\[ v_t^p = (1 - \phi_p) \left( \Pi_t^p \right)^{-\epsilon_p} + \phi_p \Pi_t^{\epsilon_p} v_{t-1}^p \] (16)

Labor market-clearing requires that labor supplied by the household equal the sum total of labor employed by unions:

Labor market clearing: labor supplied by the household must equal labor used by the union:

\[ L_t = \int_0^1 L_{u,t}(l) dl \]

Given the demand for union labor, this works out to:

\[ L_t = L_{d,t} v_t^w \] (17)

Where \( v_t^w \) is a measure of wage dispersion:

\[ v_t^w = (1 - \phi_w) \left( \frac{w_t^w}{w_t} \right)^{-\epsilon_w} + \phi_w \Pi_t^{\epsilon_w} \left( \frac{w_t}{w_{t-1}} \right)^{\epsilon_w} v_{t-1}^w \] (18)

To get the aggregate resource constraint, first aggregate dividends from retail firms

\[ DIV_{r,t} = \int_0^1 DIV_{r,t}(f) = P_t^{\epsilon_r} Y_t \int_0^1 P_t(f)^{1-\epsilon_r} df - P_{w,t} Y_t \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{-\epsilon_p} df \]

Which, given the price index and definition of price dispersion is:

\[ DIV_{R,t} = P_t Y_t - P_{w,t} Y_t v_t^p \]
Now aggregate dividends from unions:

\[
DIV_{u,t} = \int_0^1 DIV_{u,t}(l)dl = W_t^{\epsilon_w} L_{d,t} \int_0^1 W_t(l)^{1-\epsilon_w}dl - MRST_{L,d,t} \int_0^1 \left( \frac{W_t(l)}{W_t} \right)^{-\epsilon_w}dl
\]

Which given the wage index works out to:

\[
DIV_{u,t} = W_t L_{d,t} - MRST_{L,P,t} v_{t}^{w}
\]

The dividend from wholesale firm is:

\[
DIV_{W,t} = P_{w,t} Y_{w,t} - W_t L_{d,t}
\]

Total dividends received by the household are then:

\[
DIV_t = DIV_{r,t} + DIV_{w,t} + DIV_{W,t}
\]

Summing these up:

\[
DIV_t = P_t Y_t - P_{w,t} Y_{w,t} + W_t L_{d,t} - MRST_{L_d,t} v_{t}^{w} + P_{w,t} Y_{w,t} - W_t L_{W,t}
\]

But then using facts about labor and goods market-clearing, we have:

\[
DIV_t = P_t Y_t - P_{w,t} Y_{W,t} + W_t L_{d,t} - MRST_{L,t} + P_{w,t} Y_{W,t} - W_t L_{d,t}
\]

But then stuff cancels, leaving:

\[
DIV_t = P_t Y_t - MRST_{L,t}
\]

Then if we impose bonds in zero supply (which is innocuous, we could have different kinds of firms buying/selling debt and it wouldn’t affect anything), we get the standard resource constraint:

\[
Y_t = C_t
\] (19)

6 Exogenous Process

\(A_t\) is the only exogenous variable. Assume it follows an AR(1) in the log with non-stochastic mean normalized to unity:

\[
\ln A_t = \rho A \ln A_{t-1} + s_A \epsilon_{A,t}
\] (20)
7 Full Set of Equilibrium Conditions

- Household:

\[ \psi L_t^x = C_t^{-\sigma} mrs_t \]  
(21)

\[ 1 = R_t E_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \]  
(22)

\[ \Lambda_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \]  
(23)

- Wage-setting:

\[ w_t^\# = \frac{\epsilon_w f_{1,t}}{\epsilon_w - 1 f_{2,t}} \]  
(24)

\[ f_{1,t} = mrs_t w_t^w L_{d,t} + \phi_w E_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_w} f_{1,t+1} \]  
(25)

\[ f_{2,t} = w_t^w L_{d,t} + \phi_w E_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_w-1} f_{2,t+1} \]  
(26)

- Price-setting:

\[ x_{1,t} = p_{w,t} Y_t + \phi_p E_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_p} x_{1,t+1} \]  
(27)

\[ x_{2,t} = Y_t + \phi_p E_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_p-1} x_{2,t+1} \]  
(28)

\[ \Pi_t^\# = \frac{\epsilon_p}{\epsilon_p - 1} \frac{x_{1,t}}{x_{2,t}} \]  
(29)

- Wholesale firm:

\[ Y_{W,t} = A_t L_{d,t} \]  
(30)

\[ w_t = p_{w,t} A_t \]  
(31)

- Monetary policy:

\[ \ln R_t = (1 - \rho_R) \ln R + \rho_R \ln R_{t-1} + (1 - \rho_R) \theta_\pi (\ln \Pi_t - \ln \Pi) + s_R \varepsilon_{R,t} \]  
(32)

- Aggregate conditions:

\[ 1 = (1 - \phi_p) \left( \Pi_t^\# \right)^{1-\epsilon_p} + \phi_p \Pi_t^{\epsilon_p} \]  
(33)

\[ w_t^{1-\epsilon_w} = (1 - \phi_w) \left( w_t^\# \right)^{1-\epsilon_w} + \phi_w \Pi_{t-1}^{\epsilon_w} \]  
(34)

\[ Y_{W,t} = Y_t v_t^p \]  
(35)
\[ v^p_t = (1 - \phi_p) \left( \Pi^\#_t \right)^{-\epsilon_p} + \phi_p \Pi^\#_t v^p_{t-1} \]  
\[ L_t = L_{d,t} v^w_t \]  
\[ v^w_t = (1 - \phi_w) \left( \frac{w^\#_t}{w_t} \right)^{-\epsilon_w} + \phi_w \Pi^\# w_t \left( \frac{w_t}{w_{t-1}} \right)^{\epsilon_w} v^w_{t-1} \]  
\[ Y_t = C_t \]  
\[ Y_t^t = C_t \]  
\[ L_t = L_{d,t} v^w_t \]

- Exogenous process:

\[ \ln A_t = \rho_A \ln A_{t-1} + s_A \epsilon_A, t \]  

This is 20 variables \( \{ C_t, Y_t, Y^w_t, L_t, L_{d,t}, A_{t,t+1}, R_t, mrs_t, w_t, p, w^\#, w^\#, x_{1,t}, x_{2,t}, f_{1,t}, f_{2,t}, A_t, v^p_t, v^w_t \} \) in 20 equations.

8 Steady State

It is easiest to assume zero net inflation in steady state. This means \( \Pi = 1 \), which then implies \( \Pi^\# = \Pi^p = \Pi^w = 1, R = 1/\beta, \) and \( w^\# = w \). Note also we have already assumed \( A = 1 \).

From the FOC for price-setting, we get:

\[ p_w = w = \frac{\epsilon_p - 1}{\epsilon_p} \]  

From the FOC for wage-setting, we see:

\[ mrs = \frac{\epsilon_w - 1}{\epsilon_w} \]  

Combining these, we get:

\[ mrs = \frac{\epsilon_w - 1}{\epsilon_w} \frac{\epsilon_p - 1}{\epsilon_p} \]  

In an efficient allocation, we would have \( mrs = 1 \). Market-power in both labor and goods distort this, with \( mrs < 1 \).

I will calibrate the model to be consistent with \( L = 1 \) in steady state. This means that \( \psi \) must satisfy:

\[ \psi = mrs \]