Equilibrium with Production and Endogenous Labor Supply

ECON 30020: Intermediate Macroeconomics

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Spring 2018
Readings

- GLS Chapter 11
Production and Labor Supply

- We continue working with a two period, optimizing, equilibrium model of the economy.
- No uncertainty over future, although it would be straightforward to entertain this.
- We augment the model with which we have been working along the following two dimensions:
  1. We model production and an investment decision.
  2. Model endogenous labor supply.
- The production side is very similar to the Solow model.
Firm

There exists a representative firm. The firm produces output using capital, $K_t$, and labor, $N_t$, according to the following production function:

$$Y_t = A_t F(K_t, N_t)$$

- $A_t$ is exogenous productivity variable. Abstract from trend growth.

- $F(\cdot)$ has the same properties as assumed in the Solow model – increasing in both arguments, concave in both arguments, both inputs necessary. For example:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha}, \quad 0 < \alpha < 1$$
Capital Accumulation

- Slightly differently than the Solow model, we assume that the firm makes the capital accumulation decisions.
- We assume that the firm must borrow from a financial intermediary in order to finance its investment.
- “Equity” versus “debt” finance would be equivalent absent financial frictions, which we will model.
- Furthermore, ownership of capital wouldn’t make a difference absent financial frictions (i.e. firm makes capital accumulation decision vs. household owning capital and leasing it to firms).
- Current capital, $K_t$, is predetermined and hence exogenous. Capital accumulates according to:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

- Exactly same accumulation equation as in Solow model.
Prices Relevant for the Firm

- Firm hires labor in a competitive market at (real) wage $w_t$ (and $w_{t+1}$ in the future)
- Firm borrows to finance investment at:
  
  \[ r_t^I = r_t + f_t \]
  
- $r_t^I$ is the interest rate relevant for the firm, while $r_t$ is the interest rate relevant for the household
- $f_t$ is (an exogenous) variable representing a financial friction. We will refer to this as a credit spread
- During financial crises observed credit spreads rise significantly
Dividends

- The representative household owns the firm. The firm returns any difference between revenue and cost to the household each period in the form of a dividend.

- Dividend is simply output (price normalized to one since model is real) less cost of labor in period \( t \) (since borrowing cost of investment is borne in future):

\[
D_t = Y_t - w_t N_t
\]

- Terminal condition for the firm: firm wants \( K_{t+2} = 0 \) (die with no capital). This implies \( I_{t+1} = -(1 - \delta)K_{t+1} \), which we can think of as the firm “liquidating” its remaining capital after production in \( t + 1 \).

- This is an additional source of revenue for the firm in \( t + 1 \). In addition, firm has to pay interest plus principal on its borrowing for investment in \( t \):

\[
D_{t+1} = Y_{t+1} + (1 - \delta)K_{t+1} - w_{t+1} N_{t+1} - (1 + r^I_t) I_t
\]
Firm Valuation and Problem

- Value of the firm: PDV of flow of dividends:

\[ V_t = D_t + \frac{1}{1 + r_t} D_{t+1} \]

- Firm problem is to pick \( N_t \) and \( I_t \) to maximize \( V_t \) subject to accumulation equation:

\[
\max_{N_t, I_t} V_t = D_t + \frac{1}{1 + r_t} D_{t+1}
\]

s.t.

\[
K_{t+1} = I_t + (1 - \delta) K_t
\]

\[
D_t = A_t F(K_t, N_t) - w_t N_t
\]

\[
D_{t+1} = A_{t+1} F(K_{t+1}, N_{t+1}) + (1 - \delta) K_{t+1} - w_{t+1} N_{t+1} - (1 + r_t^I) I_t
\]
First Order Conditions

- Two first order conditions come out of firm problem:

\[ w_t = A_t F_N(K_t, N_t) \]

\[ 1 + r_t^I = A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta) \]

- Intuition: MB = MC
- Wage condition exactly same as Solow model expression for wage
- Investment condition can be re-written in terms of earlier notation by noting \( R_{t+1} = A_{t+1} F_K(K_{t+1}, N_{t+1}) \) and:

\[ R_{t+1} = r_t^I + \delta = r_t + f_t + \delta \]

- Return on capital, \( R_{t+1} \), closely related to real interest rate, \( r_t \)
- These FOC implicitly define labor and investment demand functions
Labor Demand

- Labor FOC implicitly characterizes a downward-sloping labor demand curve:

\[ N_t = N^d(d, A_t, K_t) \]

\[ \text{\uparrow} A_t \text{ or } \uparrow K_t \]
Second first order condition implicitly defines a demand for $K_{t+1}$, which can be used in conjunction with the accumulation equation to get an investment demand curve:

$$I_t = I_d(r_t, A_{t+1}, f_t, K_t)$$
Household

- There exists a representative household. Households get utility from consumption and leisure, where leisure is $L_t = 1 - N_t$, with $N_t$ labor and available time normalized to 1.

- Lifetime utility:

$$U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$$

- Example flow utility functions:

$$u(C_t, 1 - N_t) = \ln C_t + \theta_t \ln(1 - N_t)$$

$$u(C_t, 1 - N_t) = \ln[C_t + \theta_t \ln(1 - N_t)]$$

- Here, $\theta_t$ is an exogenous “labor supply shock” governing utility from leisure (equivalently, disutility from labor).

- Notation: $u_C$ denotes marginal utility of consumption, $u_L$ marginal utility of leisure (marginal utility of labor is $-u_L$).
Budget Constraints

- Household faces two flow budget constraints, conceptually the same as before, but now income is partly endogenous:

\[ C_t + S_t \leq w_t N_t + D_t \]
\[ C_{t+1} + S_{t+1} - S_t \leq w_{t+1} N_{t+1} + D_{t+1} + D_{t+1}^l + r_t S_t \]

- Household takes \( D_t, D_{t+1}, \) and \( D_{t+1}^l \) (dividend from financial intermediary) as given (ownership different than management)

- Terminal condition: \( S_{t+1} = 0 \). Gives rise to IBC:

\[ C_t + \frac{C_{t+1}}{1 + r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D_{t+1}^l}{1 + r_t} \]
First Order Conditions

- Do the optimization in the usual way. The following first order conditions emerge:

\[
u_C(C_t, 1 - N_t) = \beta (1 + r_t) u_C(C_{t+1}, 1 - N_{t+1})\]

- This is the usual Euler equation, only looks different to accommodate utility from leisure

\[
u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)
\]

\[
u_L(C_{t+1}, 1 - N_{t+1}) = w_{t+1} u_C(C_{t+1}, 1 - N_{t+1})\]

- Discussion and intuition
Optimal Decision Rules

- Can go from first order conditions to optimal decision rules
- Cutting a few corners, we get the same consumption function as before:
  \[ C_t = C^d(Y_t, Y_{t+1}, r_t) \]

- Or, if there were government spending, with Ricardian Equivalence we’d have:
  \[ C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \]
Labor Supply

- First order condition for $N_t$ can be characterized by an indifference curve / budget line diagram similar to the two period consumption case:

- Things are complicated for a few reasons:
  - Competing income and substitution effects of $w_t$
  - Non-wage income and expectations about future income (including through an interest rate channel) can affect current labor supply

- We will sweep most of this stuff under rug: substitution effect dominates and other things (other than exogenous variable $\theta_t$) are ignored

- Can be motivated explicitly with preference specification due to Greenwood, Hercowitz, and Huffman (1988):

$$u(C_t, 1 - N_t) = \ln \left[ C_t + \theta_t \ln(1 - N_t) \right]$$
Labor Supply Curve

- Labor supply function under these assumptions:

\[ N_t = N^s(w_t, \theta_t) \]
Financial Intermediary

- Will not go into great detail
- In period $t$, takes in deposits, $S_t$, from household; issues loans in amount $I_t$ to firm
- Pays $r_t$ for deposits, and earns $r_t^I = r_t + f_t$ on loans
- $f_t$ is exogenous, and $f_t > 0$ means intermediary earns profit in $t + 1$, which is returned to household as dividend:

$$D_{t+1}^I = (r_t + f_t)I_t - r_t S_t$$
Market-Clearing

- Market-clearing requires \( S_t = I_t \) (i.e. funds taken in by financial intermediary equal funds distributed to firm for investment)

- This implies:
  \[
  Y_t = C_t + I_t
  \]

- If there were a government levying (lump sum) taxes on household period \( t \) resource constraint would just be:
  \[
  Y_t = C_t + I_t + G_t
  \]

- Can show that period \( t + 1 \) constraint is the same:
  \[
  Y_{t+1} = C_{t+1} + I_{t+1}
  \]
Equilibrium

- The following conditions must all hold in period $t$ in equilibrium:

\[ C_t = C^d(Y_t, Y_{t+1}, r_t) \]
\[ N_t = N^s(w_t, \theta_t) \]
\[ N_t = N^d(w_t, A_t, K_t) \]
\[ I_t = I^d(r_t, A_{t+1}, f_t, K_t) \]
\[ Y_t = A_t F(K_t, N_t) \]
\[ Y_t = C_t + I_t \]

- Endogenous: $C_t, N_t, Y_t, I_t, w_t$, and $r_t$

- Exogenous: $A_t, A_{t+1}, K_t, f_t, \theta_t$. Will talk about $Y_{t+1}$ and $K_{t+1}$ later

- Four optimal decision rules, two resource constraints: income = production and income = expenditure
Competitive Equilibrium

- There are now two prices – $r_t$ (intertemporal price of goods) and $w_t$ (price of labor).

- Different ways to think about what the markets are. One is clear – market for labor, which $w_t$ adjusts to clear (i.e. labor supply = demand).

- Can think about either market for goods (i.e. $Y_t = C_t + I_t$) or a loanable funds market $S_t = I_t$ as being the other market, which $r_t$ adjusts to clear. We will focus on market for goods.

- Endowment economy special case of this if $N_t$ and $I_t$ are held fixed.

- Will be possible to do some consumption smoothing in equilibrium here, however. Suppose household wants to increase $S_t$. It can do this if $r_t$ falls to incentivize more $I_t$ (whereas in endowment economy $I_t = 0$, so $S_t$ must remain fixed at 0).