Extra Questions: Consumption

Intermediate Macroeconomics, Fall 2015
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Instructions: The following are extra questions for practice and review. The topic covered is consumption.

1. Suppose a household has the following lifetime utility function:

\[ U = C_t^{1/2} + \beta C_{t+1}^{1/2} \]

(a) Find expressions for the partial derivatives of lifetime utility, \( U \), with respect to period \( t \) and period \( t+1 \) consumption. Is marginal utility of consumption in both periods always positive?

(b) Find expressions for the second derivatives of lifetime utility with respect to period \( t \) and \( t+1 \) consumption, i.e. \( \frac{\partial^2 U}{\partial C_t^2} \) and \( \frac{\partial^2 U}{\partial C_{t+1}^2} \). Are these second derivatives always negative for any positive values of period \( t \) and \( t+1 \) consumption?

(c) Derive an expression for the indifference curve associated with lifetime utility level \( U_0 \) (i.e. derive an expression for \( C_{t+1} \) as a function of \( U_0 \) and \( C_t \)). What is the slope of the indifference curve? How does the magnitude of the slope vary with the value of \( C_t \)?

(d) Suppose that the household faces two within period budget constraints of the form:

\[ C_t + S_t = Y_t \]

\[ C_{t+1} = Y_{t+1} + (1 + r_t)S_t \]

Combine the two period budget constraints into one intertemporal budget constraint.

(e) Use the intertemporal budget constraint and this utility function to derive the Euler equation characterizing an optimal consumption plan.

(f) Use this Euler equation and the intertemporal budget constraint to derive a consumption function expressing \( C_t \) as a function of \( Y_t \), \( Y_{t+1} \), and \( r_t \)

(g) Use your consumption function to find the partial derivatives of \( C_t \) with respect to \( Y_t \), \( Y_{t+1} \), and \( r_t \).

(h) Is the MPC positive and less than 1? Does it depend on the value of \( r_t \) here?

(i) Is consumption decreasing in the real interest rate?

(j) Is consumption increasing in \( Y_{t+1} \)?

2. Now, consider a household who gets utility from period \( t \) and \( t+1 \) consumption, but where the function mapping consumption at each date into utility is not the same. In particular, assume that lifetime utility is:

\[ U = C_t + \beta \ln C_{t+1} \]

The period \( t \) and \( t+1 \) budget constraints are the same as in the previous problem, and therefore the intertemporal budget constraint is the same.
(a) Find the Euler equation for this utility function.
(b) Use this Euler equation and the intertemporal budget constraint to derive the consumption function, expressing $C_t$ as a function of $Y_t$, $Y_{t+1}$, and $r_t$.
(c) What is the MPC for this utility function?
(d) If households had this utility function, would temporary tax cuts be more or less effective than with a more standard utility function?

3. Now, suppose we have a household with the following (non-differentiable) utility function:

$$U = \min(C_t, C_{t+1})$$

With this utility function, utility equals the minimum of period $t$ and $t+1$ consumption. For example, if $C_t = 3$ and $C_{t+1} = 4$, then $U = 3$. If $C_t = 3$ and $C_{t+1} = 6$, then $U = 3$. If $C_t = 5$ and $C_{t+1} = 4$, then $U = 4$.

(a) Since this utility function is non-differentiable, you cannot use calculus to characterize optimal behavior. Instead, think about it a little bit without doing any math. What must be true about $C_t$ and $C_{t+1}$ if a household with this utility function is behaving optimally?
(b) The two within period budget constraints are the same as above. Use the condition from (a) and the intertemporal budget constraint to derive the consumption function.
(c) Is the MPC between 0 and 1? Is consumption decreasing in the real interest rate?

4. Suppose we have a household with the following lifetime utility function:

$$U = \ln C_t + \beta \ln C_{t+1}$$

The two period budget constraints are:

$$C_t + S_t = Y_t$$
$$C_{t+1} = Y_{t+1} + S_t + (1 - \tau) r_t S_t$$

Here $\tau$ is a tax on interest income (if $r_t S_t > 0$, then $\tau > 0$ means that you get to keep less of your return on saving). Derive the consumption function for this problem. How will $C_t$ be affected by an increase in $\tau$?

5. Suppose that we have a household that lives for three periods – $t$, $t+1$, and $t+2$. Its lifetime utility function is:

$$U = \ln C_t + \beta \ln C_{t+1} + \beta^2 \ln C_{t+2}$$

There are now three period budget constraints, given by:

$$C_t + S_t = Y_t$$
$$C_{t+1} + S_{t+1} = Y_t + (1 + r_t) S_t$$
$$C_{t+2} + S_{t+2} = Y_{t+1} + (1 + r_{t+1}) S_{t+1}$$

(a) What must be true about $S_{t+2}$? Why?
(b) Use this condition on $S_{t+2}$ to combine the three period budget constraints into one intertemporal budget constraint.
(c) Characterize the optimum. Hint: you solve for $C_{t+2}$ in terms of $C_{t+1}$ and $C_t$. Plug this in to the lifetime utility function. Then find the FOC with respect to $C_t$ and $C_{t+1}$. You will be left with two Euler equations.

(d) Combine the two Euler equations with the intertemporal budget constraint to solve for the consumption function, expressing $C_t$ as a function of $Y_t$, $Y_{t+1}$, $Y_{t+2}$, $r_t$, and $r_{t+1}$.

(e) Is the MPC for this specification bigger or smaller than what it would if the household only lives for two periods? What is the intuition for this?

(f) What is the sign of the effect of $r_{t+1}$ on $C_t$?

6. Suppose that a household lives for $T+1$ periods. Its lifetime utility function is:

$$U = \sum_{j=0}^{T} \beta^j u(C_{t+j})$$

Assume that the real interest rate is constant across time. The intertemporal budget constraint is therefore:

$$\sum_{j=0}^{T} \frac{C_{t+j}}{(1+r)^j} = \sum_{j=0}^{T} \frac{Y_{t+j}}{(1+r)^j}$$

(a) What will the Euler equations for each two adjacent periods of time look like?

(b) Assuming that $u''(\cdot) < 0$, what will the time path of consumption look like for this household if $\beta(1+r) = 1$, $\beta(1+r) > 1$, and $\beta(1+r) < 1$? What is the intuition for this?

(c) Suppose that $\beta(1+r) = 1$. Derive an analytic expression $C_t$.

(d) What is the marginal propensity to consume?

(e) Suppose that income grows over time, obeying the process: $Y_{t+j} = (1+g_Y)^j Y_t$. Assume that $T$ is sufficiently big so that $\sum_{j=0}^{T} \alpha^j = \frac{1}{1-\alpha}$. Assume that $g_Y < r$. Derive an expression for $C_t$ that depends only on $\beta$, $g_Y$, and $Y_t$. Is the MPC under this assumption about income growth bigger or smaller than what you found above? What is the intuition for your answer?

(f) How should period $t$ saving, $Y_t - C_t$, react to an increase in $g_Y$? What is the intuition for your answer?

7. Suppose that a household lives for two periods, $t$ and $t+1$. Its lifetime utility function is:

$$U = u(C_t) + \beta u(C_{t+1})$$

The period utility function $u(\cdot)$ is increasing and concave. The household faces two period budget constraints:

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1 + r_t)$$

In addition, the household faces a liquidity constraint that its period $t$ saving cannot be negative (i.e. it cannot borrow):

$$S_t \geq 0$$

(a) What does it mean for the borrowing constraint to “bind”?
(b) Is the borrowing constraint more likely to bind if $Y_{t+1} > Y_t$ or if $Y_t > Y_{t+1}$? What is the intuition for this?

(c) Given the normal life cycle income pattern (it tends to grow), for what demographic group is a borrowing constraint most likely to bind?

(d) Use an indifference curve / budget line diagram to argue that the MPC out of an increase in $Y_t$ for a household facing a binding borrowing constraint ought to be 1 (assuming that the change in income is not so big that the borrowing constraint ceases to bind).

(e) Given your answers to previous problems, for which demographic group are temporary tax cuts likely to be most effective at simulating consumption?