Instructions: The following are extra questions for practice and review. The topic covered is equilibrium in an endowment economy.

1. Suppose that there exist many identical households in an economy. The representative household has the following lifetime utility function:

$$U = \ln C_t + \beta \ln C_{t+1}$$

It faces a sequence of period budget constraints which can be combined into one intertemporal budget constraint:

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

The endowment, $Y_t$ and $Y_{t+1}$, is exogenous, and the household takes the real interest rate as given.

(a) Derive the consumption function for the representative household.

(b) Derive a saving function for this household, where saving is defined as $S_t = Y_t - C_t$ (plug in your consumption function and simplify).

(c) Since the economy is populated by many identical households, what must be true about $S_t$ in equilibrium? Why?

(d) Write down the definition of the $Y^d$ curve. Derive an analytic expression for the $Y^d$ curve with the consumption function you found above.

(e) Solve for expressions for the equilibrium values of $r_t$, $Y_t$, and $Y_{t+1}$.

(f) Use these expressions to determine how $r_t$ reacts to changes in $Y_t$ and $Y_{t+1}$ (in isolation, not together).

2. Suppose that there is an endowment economy with two types of agents – 1 and 2, which I denoted with the subscript $j$. There are $N_1$ of type 1 agents and $N_2$ of type 2 agents. $N_1$ and $N_2$ are sufficiently large that both types of agents take the real interest rate as given.

Each type of agent has identical preferences given by:

$$U_j = \ln C_{j,t} + \beta \ln C_{j,t+1}$$

Each agent faces two identical period budget constraints:

$$C_{j,t} + S_{j,t} = Y_{j,t}$$
$$C_{j,t+1} = Y_{j,t+1} + (1 + r_t)S_{j,t}$$
(a) Derive the consumption function for agent of type $j$ (hint, the consumption function will be the same for both types).

(b) Suppose that the agents differ in terms of their endowment patterns. In particular, type 1 agents have 1 unit of the fruit in period $t$ and none in period $t+1$, whereas type 2 agents have 0 units of fruit in period $t$ and 1 in period $t+1$. In other words, $(Y_{1,t}, Y_{1,t+1}) = (1, 0)$ and $(Y_{2,t}, Y_{2,t+1}) = (0, 1)$. Which type of agent would like to borrow in period $t$ and which type would like to save? Why?

(c) What will have to be true about aggregate saving in equilibrium (aggregate saving would be $N_1 S_{1,t} + N_2 S_{2,t}$)?

(d) Use this condition to solve for the equilibrium value of $r_t$ (note, it will be a function of $N_1$ and $N_2$), as well as the equilibrium allocations of consumption for both types of agents in both periods.

(e) How does the equilibrium value of $r_t$ depend on $N_1$ and $N_2$? What is the intuition for this?

(f) Suppose that there is an increase in $N_2$. How does this affect the equilibrium value of $r_t$ and the consumption allocations in both periods for both types of agents?

(g) Are type 1 agents better off or worse off (in terms of the value of $U$) after an increase in $N_2$? What about type 2 agents? What is the intuition for your answer?

3. Suppose that we have an economy with many identical households. There is a government that exogenously consumes some output and pays for it with lump sum taxes. Lifetime utility for a household is:

$$U = \ln C_t + \beta \ln C_{t+1}$$

The household faces two within period budget constraints given by:

$$C_t + S_t = Y_t - T_t$$

$$C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t$$

(a) Combine the two budget constraints into one intertemporal budget constraint.

(b) Use this to find the Euler equation. Is the Euler equation at all affected by the presence of taxes, $T_t$ and $T_{t+1}$?

(c) Use the Euler equation and intertemporal budget constraint to derive an expression for the consumption function.

The government faces two within period budget constraints:

$$G_t + S^G_t = T_t$$

$$G_{t+1} = T_{t+1} + (1 + r_t)S^G_t$$

(d) In equilibrium, what must be true about $S_t$ and $S^G_t$?

(e) Combine the two period budget constraints for the government into one intertemporal budget constraint.

(f) Suppose that the representative household knows that the government’s intertemporal budget constraint must hold. Combine this information with the household’s consumption function you derived above. What happens to $T_t$ and $T_{t+1}$? What is your intuition for this?
(g) Equilibrium requires that \( Y_t = C_t + G_t \). Plug in your expression for the consumption function (assuming that the household knows the government’s intertemporal budget constraint must hold) to derive an expression for \( Y_t \).

(h) Derive an expression for the “fixed interest rate multiplier,” i.e. \( \frac{dY_t}{dG_t} |_{dr_t=0} \).

(i) Assuming that \( Y_t \) is exogenous, what must happen to \( r_t \) after an increase in \( G_t \)?

(j) Now, assume the same setup but suppose that the household does not anticipate that the government’s intertemporal budget constraint will hold – in other words, do not combine the government’s intertemporal budget constraint with the household’s consumption function as you did on part (f). Repeat part (h), deriving an expression for the “fixed interest rate multiplier” while not assuming that the household anticipates the government’s budget constraint holding. Is it bigger or smaller than you found in (h)?

(k) Since \( Y_t \) is exogenous, what must happen to \( r_t \) after an increase in \( G_t \) in this setup? Will the change in \( r_t \) be bigger or smaller here than what you found in part (i)?

(l) For the setup in which the household does not anticipate that the government’s intertemporal budget constraint must hold, what will be the “fixed interest rate tax multiplier”, i.e. \( \frac{dY_t}{dT_t} |_{dr_t=0} \)? Is this different than what the tax multiplier would be if the household were to anticipate that the government’s intertemporal budget constraint must bind? Is it smaller or larger than the fixed interest rate multiplier for government spending (assuming that the household does not anticipate that the government’s intertemporal budget constraint will hold)?