Extra Questions: Neoclassical Model

Intermediate Macroeconomics, Fall 2015
The University of Notre Dame
Professor Sims

Instructions: The following are extra questions for practice and review. The topic covered is the neoclassical business cycle model.

1. Suppose that we have a firm who owns its capital stock and hires labor each period. Output is produced according to:

\[ Y_t = A_t K_t^\alpha N_t^{1-\alpha} \]

Firm profit in period \( t \) is:

\[ \Pi_t = Y_t - w_t N_t - I_t \]

In period \( t + 1 \) profit is:

\[ \Pi_{t+1} = Y_{t+1} - w_{t+1} N_{t+1} - I_{t+1} \]

The capital accumulation equation is:

\[ K_{t+1} = q I_t + (1 - \delta) K_t \]

The capital accumulation equation for period \( t + 1 \) is:

\[ K_{t+2} = q I_{t+1} + (1 - \delta) K_{t+1} \]

The firm exists for two periods, \( t \) and \( t + 1 \). Its capital stock in period \( t \), \( K_t \), is exogenous. The value of the firm is the present discounted value of profit, where discounting is by \( \frac{1}{1+r_t} \):

\[ V_t = \Pi_t + \frac{1}{1+r_t} \Pi_{t+1} \]

(a) Since the firm “dies” after period \( t + 1 \), what will it want \( K_{t+2} \) to be? What does this imply about \( I_{t+1} \)? What is the economic interpretation of this value of \( I_{t+1} \)?

(b) Impose this condition from (a) and use the capital accumulation equation in period \( t \) and production function to write the value of the firm in terms of \( N_t, N_{t+1}, K_t, \) and \( K_{t+1} \), plus other exogenous variables and parameters. Find the first order conditions characterizing firm optimality (the firm can choose \( N_t, N_{t+1}, \) and \( K_{t+1} \)). Provide an intuitive interpretation of each condition.

(c) Use your first order condition for \( K_{t+1} \), along with the capital accumulation equation, to argue that the demand for current investment, \( I_t \), is a decreasing function of the real interest rate, \( r_t \).

2. Now let’s consider a model in which the household makes capital accumulation decisions and leases capital each period to firms. The firm problem is the same as in the Solow model. Its problem is now static (since the firm doesn’t own its own capital, there is nothing inherently dynamic about the choices the firm makes). Profit in each period is:

\[ \Pi_t = A_t K_t^\alpha N_t^{1-\alpha} - w_t N_t - R_t K_t \]

\[ \Pi_{t+1} = A_{t+1} K_{t+1}^\alpha N_{t+1}^{1-\alpha} - w_{t+1} N_{t+1} - R_{t+1} K_{t+1} \]
(a) Each period, the firm can choose capital and labor to maximize profit. Find the first order conditions characterizing profit-maximizing behavior by the firm.

Now, suppose that the household owns the capital stock. Its lifetime utility function is:

$$U = \ln C_t + \ln(1 - N_t) + \beta \ln C_{t+1} + \beta \ln(1 - N_{t+1})$$

The household’s budget constraint in the first period is:

$$C_t + I_t + S_t = w_t N_t + R_t K_t + \Pi_t$$

In the second period the household’s budget constraint is:

$$C_{t+1} + I_{t+1} + S_{t+1} = w_{t+1} N_{t+1} + R_{t+1} K_{t+1} + \Pi_{t+1} + (1 + r_t) S_t$$

The capital accumulation equation in period $t$ is:

$$K_{t+1} = q I_t + (1 - \delta) K_t$$

In period $t + 1$ it is:

$$K_{t+2} = q I_{t+1} + (1 - \delta) K_{t+1}$$

(b) Since the household “dies” after period $t + 1$, what will it want $S_{t+1}$ and $K_{t+2}$ to equal? Why?

(c) Use these facts to write the second period budget constraint without reference to $S_{t+1}$ and in terms of $K_{t+1}$ (instead of $I_{t+1}$, which you can substitute out after imposing the terminal condition on $K_{t+2}$).

(d) Eliminate $I_t$ from the first period budget constraint by solving for $I_t$ in terms of $K_t$ and $K_{t+1}$ from the period $t$ capital accumulation equation.

(e) Solve for $S_t$ in the revised second period budget constraint. Plug this into the first period budget constraint to produce a single unified intertemporal budget constraint.

(f) Solve for $C_{t+1}$ in terms of other variables in the revised intertemporal budget constraint. Plug this in for $C_{t+1}$ in the lifetime utility function. The remaining choice variables for the household are $N_t$, $C_t$, $C_{t+1}$, and $K_{t+1}$ and the household optimization problem should now be unconstrained. Find the first order conditions characterizing optimal behavior by the household.

(g) Combine the first order condition to derive an expression relating $R_{t+1}$ to $r_t$. Provide some intuition for this condition. Discuss this condition in light of a claim I made in the Solow model that the return on capital is closely related to the real interest rate.

(h) Combine the firm’s optimality condition for $K_{t+1}$ with the household’s optimality condition for $K_{t+1}$. Argue that this combined condition is identical to the firm’s optimality condition for $K_{t+1}$ from the problem above when the firm owns the capital stock and makes capital accumulation decisions. Argue that this means either ownership structure for the capital stock will result in the same equilibrium dynamics.

3. Write down the six mathematical conditions characterizing the equilibrium of the neoclassical model. Provide a brief written explanation for each of these equations (these equations are found on pg. 14 of the notes).
4. Graphically derive the \( Y^d \) curve in the basic neoclassical model. Which of the mathematical equations from the previous part does the \( Y^d \) curve describe?

5. Graphically derive the \( Y^s \) curve in the basic neoclassical model. Which of the mathematical equations from the previous part does the \( Y^s \) curve describe?

6. Will the \( Y^d \) curve be flatter or steeper if consumption and investment are more sensitive to the real interest rate? Why? Explain in the context of your graphical analysis of the \( Y^d \) curve.

7. Suppose that there is an exogenous reduction in \( A_t \). Graphically analyze how this will impact the values of the endogenous variables.

8. In the data, \( N_t \) moves a lot over the business cycle whereas \( w_t \) does not (though they are both procyclical, \( N_t \) is much more volatile than \( w_t \)). In the context of your answer on the previous part concerning the effects of an exogenous reduction in \( A_t \), would a flatter (more elastic) labor supply curve make the movements in \( N_t \) and \( w_t \) better fit the data, or would it make the fit worse? Explain.

9. In the basic neoclassical model, an exogenous reduction in \( A_t \) results in an increase in \( r_t \). In the data, the real interest is countercyclical but not very strongly so. Would a persistent reduction in \( A_t \) (a reduction in \( A_t \) occurring simultaneously with an expected reduction in \( A_{t+1} \)) potentially allow the model to better account for the behavior of the real interest rate?

10. Graphically and mathematically analyze the consequences of an exogenous increase in \( q \) in the model.

11. Graphically and mathematically analyze the consequences of an exogenous decrease in \( A_{t+1} \) in the model.

12. Suppose that the economy is hit by a hurricane, and some of its current capital stock, \( K_t \), is exogenously destroyed. Graphically analyze how this will impact the economy. Can you determine whether investment will go up or down with certainty? What about consumption? What will happen to the real wage, employment, and output? Note: this question is a bit tricky, as both the \( Y^d \) and \( Y^s \) curves will shift together.

13. Briefly describe (in words) how we can show that the equilibrium of the neoclassical model is efficient, and explain what is meant by efficiency. Do agents in the model like it when \( A_t \) goes down? Should a government do anything in response to \( A_t \) decreasing?

14. Write down several possible criticisms of the neoclassical business cycle model. Which one do you think has the most merit? Which one the least? There is no right or wrong answer here, I just want you to think critically about the model.