1. Money

(a) We give money a functional definition – money is something which serves as a medium of exchange, a store of value, and a unit of account

(b) Fiat money is money which has no inherent value (e.g. paper money, electronic money), but has value in exchange because a government has declared that it does (by fiat) and agents accept it in exchange

(c) New variables: \( M_t \): quantity of money; \( P_t \): price of goods in terms of money; \( i_t \): nominal interest rate

(d) Money enters model as means by which to transfer resources across time (store of value).
   Period \( t \) and \( t + 1 \) budget constraints:
   \[
P_t C_t + P_t S_t + M_t = P_t w_t N_t - P_t T_t + P_t \Pi_t
   \]
   \[
P_{t+1} C_{t+1} = P_{t+1} w_{t+1} - P_{t+1} T_{t+1} + P_{t+1} \Pi_{t+1} + (1 + i_t) P_t S_t + M_t
   \]

(e) If you look at these, money enters the budget constraint essentially identically to \( P_t S_t \) (nominal bond-holdings, \( S_t \) is real bond-holding and multiplication by \( P_t \) puts it in nominal terms). The difference is that money pays no interest, whereas bonds pay \( i_t \)

(f) Fisher relationship: connection between real and nominal interest rates. Given by:
   \[
   1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}^e}
   \]

(g) Expected inflation is defined as \( 1 + \pi_{t+1}^e = \frac{P_{t+1}}{P_t} \) – it is expected inflation between \( t \) and \( t + 1 \). For simplicity, we take this to be exogenously given. The Fisher relationship can be written in approximate terms as simple the real interest rate equals the nominal interest rate less the expected rate of inflation:
   \[
   r_t = i_t - \pi_{t+1}^e
   \]

(h) Using the Fisher relationship, combine the period budget constraints into one real intertemporal budget constraint:
   \[
   C_t + \frac{C_{t+1}}{1 + r_t} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t} = w_t N_t - T_t + \Pi_t + \frac{w_{t+1} N_{t+1} - T_{t+1} + \Pi_{t+1}}{1 + r_t}
   \]

(i) This looks the same as we’ve had before, except we have a new term – \( \frac{i_t M_t}{P_t} \). \( \frac{M_t}{P_t} \) is real money balances – it says how many goods you could purchase in period \( t \) with the money you hold between \( t \) and \( t + 1 \).
The really important function of money is as a medium of exchange, which solves the double coincidence of wants problem associated with barter. In a single good model like ours, this is not a problem and so there really isn’t an important medium of exchange component. Since money essentially just enters the model as a store of value, and it’s a crummy store of value in the sense of paying no interest (unlike bonds), it is difficult to get people to want to hold money. So we take a short cut. We assume that households get utility from holding real money balances. The idea here is that the more purchasing power of the money you hold, the easier is it for you to conduct transactions, and therefore the higher is your utility. We write lifetime utility as:

\[ U = u(C_t) + v(1 - N_t) + \phi \left( \frac{M_t}{P_t} \right) + \beta u(C_{t+1}) + \beta v(1 - N_{t+1}) \]

\( \phi(\cdot) \) is an increasing and concave function, e.g. \( \ln(\cdot) \). Even though money is held across periods, we assume that you get utility from money in period \( t \).

The household’s problem is to pick current and future consumption, current and future labor supply, and holdings of money to maximize lifetime utility subject to the intertemporal budget constraint. The first order conditions for consumption and labor supply are identical to what we have seen before. The new first order condition is:

\[ \frac{\partial U}{\partial M_t} = 0 \Leftrightarrow \phi' \left( \frac{M_t}{P_t} \right) \frac{1}{P_t} - \beta \frac{i_t}{1 + i_t} \frac{1}{P_t}(1 + r_t)u'(C_{t+1}) = 0 \]

This can be re-arranged to yield:

\[ \phi' \left( \frac{M_t}{P_t} \right) = \frac{i_t}{1 + i_t} u'(C_t) \]

From this condition, using facts about the second derivatives of the functions \( \phi(\cdot) \) and \( u(\cdot) \), we can see that desired \( M_t \) is decreasing in \( i_t \), increasing in \( C_t \), and increasing in \( P_t \). In other words, you want less money the higher is the nominal interest rate (the opportunity cost of holding money, since \( i_t \) is the return on holding bonds, which are the alternative “asset” to money), increasing in the price of goods, \( P_t \), and increasing in the number of goods you are purchasing, \( C_t \).

We will take this condition and write our optimal decision rule for the amount of money to hold as:

\[ M_t = P_t M^d(r_t + \pi_{t+1}^e, Y_t) \]

Here we use the Fisher relationship, \( i_t = r_t + \pi_{t+1}^e \), to write the demand for money in terms of the real interest rate and the rate of expected inflation. We also write the demand for money in terms of \( Y_t \), instead of \( C_t \) as we ought from the first order condition derived above. This is done for ease of exposition and does not really impact any of our subsequent analysis.

We plot the money demand curve as upward-sloping in a graph with \( M_t \) on the horizontal axis and \( P_t \) on the vertical axis. That it is upward-sloping reflects the fact that, in a sense, \( \frac{1}{P_t} \) is the price of money in terms of goods. The position of the money demand curve depends on the level of \( Y_t \), the level of \( r_t \), and the amount of expected inflation. Changes in any of these variables would cause the money demand curve to shift.
(q) We assume that money supply is set exogenously by a central bank. For given values of $r_t$, $Y_t$, and $\pi_{t+1}^e$, the intersection of money demand and supply would determine the price level:

$$P_t = P_t M_d(r_t + \pi_{t+1}^e, Y_t)$$

(r) The firm and government side of the model are unaffected, other than the fact that the government effectively earns some revenue through its printing of money. The equilib-
rium conditions of the model are:

\[ N_t = N^d(w_t, A_t, K_t) \]
\[ N_t = N^s(w_t, H_t) \]
\[ C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \]
\[ I_t = I(r_t, A_{t+1}, q, K_t) \]
\[ Y_t = C_t + I_t + G_t \]
\[ Y_t = A_t F(K_t, N_t) \]
\[ M_t = P_t M^d(r_t, \pi_{t+1}^e, Y_t) \]
\[ i_t = r_t + \pi_{t+1}^e \]

(s) This is eight equations in eight endogenous variables – the real endogenous variables \( Y_t, C_t, I_t, N_t, w_t, \) and \( r_t, \) and the nominal endogenous variables \( P_t \) and \( i_t. \) The exogenous variables are \( A_t, A_{t+1}, G_t, G_{t+1}, q, K_t, \) and now also \( \pi_{t+1}^e \) and \( M_t. \) The important thing to note here is that the first six of these equations are identical to what we had in the neoclassical model. This means that the equilibrium values of real endogenous variables can be determined independent of nominal variables. This is what is meant by the “classical dichotomy” – real variables are determined independently of nominal variables (the converse is not true). This means we can use the same graphical setup as before to analyze the equilibrium of the real side of the model. And then once we know \( r_t \) and \( Y_t, \) we can determine \( P_t \) using the money market equilibrium graph.

(t) An implication of the classical dichotomy is that money is “neutral” in the sense that changes in \( M_t \) have no effect on the values of any of the real endogenous variables. An increase in the money supply just causes the price level to increase:

(u) A positive supply shock (either an increase in \( A_t \) or \( H_t \)) causes \( Y_t \) to rise and \( r_t \) to fall on the real side. These effects induce the money demand curve to pivot out to the right, resulting in a lower price level in equilibrium:
A positive demand shock to the IS curve causes $r_t$ to increase and nothing to happen to $Y_t$ on the real side. This causes the money demand curve to pivot in, and therefore the price level to rise:

\[ P_t = P_t M^d(r_t^0, Y_t^0) \]
\[ P_t^0 = P_t M^d(r_t^1, Y_t^0) \]
\[ P_t^1 = P_t M^d(r_t^0, Y_t^0) \]

What determines the average rates of inflation and nominal interest rates? In the long run, making a specific functional form assumption on money demand (log preferences over both consumption and real balances), the following expression ought to be approximately true:

\[ \pi = g^M - g^Y \]

In other words, the long run inflation rate ought to equal the difference between the average growth rate of the money supply and the growth rate of output. This approximation
holds quite well in the data

(y) The real interest rate in the long run ought to be determined by how households discount the future ($\beta$) and how fast the economy grows – it ought to be independent of anything nominal. We would naturally expect expected inflation to be equal to average realized inflation over the long run – i.e. expectations are right on average. This means that $\pi_{t+1} = \pi$. From the Fisher relationship, then, we would expect the average nominal interest rate to equal:

$$i = r + \pi$$

In other words, the average value of the interest rate ought to be determined by the inflation rate (which is in turn determined by the rate of excess growth of money over output). This also holds well in the data.

(z) Our model has the implication that money is neutral in the sense of being irrelevant for the determination of real endogenous variables even in the short run. Looking at correlations between money and output at different leads and lags, this does not seem to be the case – money and output are positively correlated, and this correlation is stronger when output is led relative to the money supply. This is suggestive evidence of monetary non-neutrality.

2. Keynesian models:

(a) Keynesian models differ from their neoclassical counterpart in featuring some form of nominal rigidity. We consider two cases – price and wage stickiness. Relative to the neoclassical model, Keynesian models have the following implications:

i. Money is non-neutral – changes in the supply of money will affect real endogenous variables

ii. Demand shocks will matter – output will not solely be supply-determined

iii. Supply shocks will have different effects on real endogenous variables than in the neoclassical model

iv. The equilibrium of the Keynesian model will not necessarily coincide with the equilibrium of the neoclassical model. This will have the implication that there is some justification for activist stabilization policies, which is different than the neoclassical model, where the equilibrium is efficient.

(b) It is useful to think about their being three different “runs” in macro corresponding to different time frequencies. The long run studies behavior at the decadal (or more) frequencies, and focusing on productivity growth and capital accumulation. We use the Solow model to study it. The medium run focuses on behavior at the frequency of several years. Over that time horizon we can think about the capital stock as roughly fixed, though we do analyze how investment reacts to shocks. We use the neoclassical model to study the medium run. The short run is frequencies less than several years. Relative to the neoclassical model, prices and/or wages are imperfectly flexible. We use the Keynesian model to study the short run. Neoclassicals and Keynesians differ along two dimensions – how long it takes to get from the short run to the medium run, and how quantitatively important price and wage stickiness are. Neoclassicals think that price and/or wage stickiness (i.e the aggregate supply curve is steep if not vertical) are not that important and that the economy quickly transitions to the medium run. Keynesians think that that price and/or wage stickiness are important (i.e. the aggregate supply
curve is relatively flat) and that it might take a long time to transition to the medium run.

(c) We begin by analyzing the demand block of the Keynesian model. It is no different than the demand block of the neoclassical model. The equations summarizing the demand block of the model are:

\[
C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \\
I_t = I(r_t, A_{t+1}, q, K_t) \\
Y_t = C_t + I_t + G_t \\
M_t = P_t M^d(r_t + \pi_{t+1} e, Y_t)
\]

These same equations show up in the equilibrium conditions of the neoclassical model. We use slightly different curves to analyze them graphically. The IS Curve is defined as the set of \((r_t, Y_t)\) pairs where households and firms are behaving optimally and income is equal to expenditure. This is exactly the same as what we called the \(Y^d\) curve before. It summarizes the first three of the above equations. It can be derived graphically:

The IS curve will shift to the right if \(A_{t+1}, q\), or \(G_t\) increase, it will shift right if \(K_t\) decreases, and it will shift left if \(G_{t+1}\) increases.

(d) The LM curve is defined as the set of \((r_t, Y_t)\) pairs where the money market is in equilibrium for given values of \(P_t\) and \(M_t\). In other words, it traces out the combination of \((r_t, Y_t)\) pairs where the last equation holds for given values of \(M_t\) and \(P_t\). It is derived graphically as follows:
The LM curve will shift to the right if $M_t$ increases, if $P_t$ decreases, or if $\pi_{t+1}$ increases.

(e) The aggregate demand (AD) curve is defined as the set of $(P_t, Y_t)$ pairs where we are on both the IS and LM curves – i.e. the combinations of $P_t$ and $Y_t$ where all of the above equations hold. It can be derived as follows:
The AD curve will shift to the right if $M_t$ or $\pi_{t+1}$ increase (which shift the LM curve) or to the right if $A_{t+1}$, $q$, or $G_t$ increase, or if $G_{t+1}$ or $K_t$ decrease (which shift the IS curve).

(f) The aggregate supply (AS curve) is defined as the set of $(P_t, Y_t)$ pairs consistent with the production function and some notion of labor market equilibrium. We can define it for the neoclassical as well as the sticky price and sticky wage Keynesian models. In the neoclassical model, the equations summarizing the supply side of the economy are:

$$N_t = N^s(w_t, H_t)$$
$$N_t = N^d(w_t, A_t, K_t)$$
$$Y_t = A_tF(K_t, N_t)$$

Intuitively, $P_t$ does not appear in any of these equations, and so there will be no relationship between $P_t$ and $Y_t$ on the supply side – i.e. the AS curve will be vertical. Graphically, we can derive it as follows:
Even though I have drawn the figure with the labor supply curve present, this is not relevant for the determination of $N_t$ or $w_t$ in equilibrium. It is simply shown for point of comparison with the neoclassical model. The full equilibrium of the model is characterized graphically as:
Even though this is a new set of graphs, there are exactly the same equations underlying these graphs as in the neoclassical model. To analyze the effects of changes in exogenous variables graphically, proceed roughly as follows:

i. Figure out whether the shock is coming on the supply side \((A_t, H_t, K_t)\) or the demand side \((A_{t+1}, q, G_t, G_{t+1}, K_t, M_t, \pi^e_{t+1})\).

ii. If on the supply side, figure out how the labor demand and/or supply curves shift, how the production function shifts, and therefore how the AS curve shifts. If on the demand side, figure out how the IS and/or LM curves shift (for a given price level),
and therefore how the AD curve shifts.

iii. Combine the AD and AS shifts to determine the new $Y_t$ and $P_t$. The new $P_t$ will imply a shift of the LM curve which makes the IS and LM curves intersect at the same level of $Y_t$ where the AS and AD curves intersect. To figure out what happens to $C_t$ and $I_t$, consider the shock under considerations as well as the effects on $r_t$ and $Y_t$.

(g) The sticky wage Keynesian model replaces the labor supply curve with the following:

$$w_t = \frac{W}{P_t}$$

Here $W$ is an exogenous nominal wage which is assumed to be fixed within period. We assume that labor is determined from the labor demand curve at the real wage implied by this nominal wage and price level. The other equations of the supply side of the model (labor demand and the production function) are the same as in the neoclassical model. This assumption of a sticky nominal wage will generate an upward-sloping AS curve. The reasoning is as follows. As the price level rises, the real wage declines for a given nominal wage, which induces firms to hire more labor. More labor means more output from the production function. The graphical derivation of the sticky wage AS curve is as follows:
Changes in $A_t$ or $K_t$ would shift the sticky wage AS curve. Since we are not on the labor supply curve, $H_t$ is irrelevant for the position of the AS curve. The demand block of the economy is identical to the neoclassical model.

(h) For the sticky price Keynesian model we assume in the background that there are many firms who produce slightly different products. The demand for product of firm $j$ depends on the relative price of firm $j$, $\frac{P_{j,t}}{P_t}$, along with other things. We assume that some fraction of firms are unable to adjust their individual prices within period to changing conditions. Firms will set their individual prices with a common expectation of the realization of the aggregate price level, $P_t^e$, which we take to be exogenous. If the aggregate price level ends up higher than firms expected, $P_t > P_t^e$, then some firms who cannot adjust their individual prices will have relative prices which are lower than they wanted. The reverse will be true if the aggregate price level is lower than anticipated. We assume that the rules of the game are that firms must produce sufficient output so as to meet demand at their relative price. Hence, if the aggregate price level is higher than anticipated, some firms will produce more than they would otherwise like, and aggregate output will be relatively high. The reverse will be true if the aggregate price level is lower than firms anticipated. We write down the following explicit equation for the aggregate supply curve:

$$P_t = P_t^e + \gamma(Y_t - Y_t^f)$$

Here $Y_t^f$ is the hypothetical amount of output which would be produced if prices were flexible $-$ i.e. this is how we will denote the hypothetical equilibrium level of output in the neoclassical model. $\gamma$ is a non-negative parameter which measures "how sticky" prices are. If $\gamma \to \infty$, all firms are able to adjust their prices and $Y_t = Y_t^f$ whether $P_t = P_t^e$ or not. You can think about $\gamma \to \infty$ as reverting to the vertical neoclassical AS curve. Small values of $\gamma$ correspond to high values of price stickiness. Graphically, the AS curve will look like:

(i) In this model, we essentially replace the labor demand curve with the AS relationship shown above. The three equations characterizing the supply side of the economy are
then:

\[ N_t = N^*(w_t, H_t) \]
\[ Y_t = A_t F(K_t, N_t) \]
\[ P_t = P_t^e + \gamma(Y_t - Y_t^f) \]

In this model, we are on the labor supply curve and off the labor demand curve. In the sticky wage model, we are on the labor demand curve and off the labor supply curve. In the neoclassical model we are on both labor demand and supply curves. Graphically, the full equilibrium of the sticky price Keynesian model is. In drawing this I draw in both labor demand and supply, even though the labor demand curve is here not relevant for the determination of \( N_t \) or \( w_t \).
(j) In either the sticky wage or sticky price models, use the following rough approach to graphically analyze the effects of changes in exogenous variables.

i. Figure out whether the stock in question is on the supply side \((A_t, K_t, H_t, W)\) in the case of the sticky wage model, or \(P_t^e\) in the case of the sticky price model) or the demand side \((A_{t+1}, q, K_t, G_t, G_{t+1}, M_t, \text{ or } \pi_{t+1}^e)\).

ii. In on the supply side, in the case of the sticky wage model, figure out how the labor demand curve and production function shift, and determine how \(Y_t\) would change for a given \(P_t\) to derive the shift in the AS curve. If on the supply side in the sticky
price model, figure out how \( Y_t^f \) changes (i.e. how the equilibrium value of \( Y_t \) would change in the neoclassical model). Graphically, this can be done by finding the \( N_t \) consistent with being on both labor demand and supply, then plugging this into the production function, and then determining the value of \( Y_t^f \). The sticky price AS curve will shift horizontally by the amount of the change in \( Y_t^f \). If on the demand side, figure out how the IS and LM curves would shift for a given \( P_t \) to determine how the AD curve would shift.

iii. Combine the AD and AS shifts to determine the new \( Y_t \) and \( P_t \). The new \( P_t \) will imply a shift of the LM curve which makes the IS and LM curves intersect at the same level of \( Y_t \) where the AS and AD curves intersect. To figure out what happens to \( C_t \) and \( L_t \), consider which exogenous variable is changing, and also look at how \( r_t \) and \( Y_t \) change. To figure out \( N_t \) and \( w_t \), work your way back to the labor market, and finding the \( N_t \) consistent with the \( Y_t \) from the intersection of the AD-AS curves. You determine the real wage by reading off the labor demand curve at this \( N_t \) in the sticky wage model, and off the labor supply curve in the sticky price model.

(k) The following table summarizes the qualitative effects of changes in exogenous variables on the endogenous variables in the sticky wage model:

<table>
<thead>
<tr>
<th>Variable:</th>
<th>↑ ( M_t )</th>
<th>↑ ( A_t )</th>
<th>↑ ( A_{t+1} )</th>
<th>↑ ( q )</th>
<th>↑ ( G_t )</th>
<th>↑ ( G_{t+1} )</th>
<th>↑ ( H_t )</th>
</tr>
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<tbody>
<tr>
<td>Output</td>
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<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Hours</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Consumption</td>
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<td>?</td>
<td>?</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Investment</td>
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<td>+</td>
<td>+</td>
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<td></td>
</tr>
<tr>
<td>Real interest rate</td>
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<tr>
<td>Real wage</td>
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<td>-</td>
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<tr>
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<td>+</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

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<th>↑ ( G_{t+1} )</th>
<th>↑ ( H_t )</th>
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<tbody>
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<td>Output</td>
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<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Hours</td>
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<td>+</td>
<td>+</td>
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<td>+</td>
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<tr>
<td>Consumption</td>
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<td>?</td>
<td>?</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Investment</td>
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<td></td>
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<tr>
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<tr>
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<tr>
<td>Price level</td>
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<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

(m) Relative to the neoclassical model, output will react less to supply shocks in the sticky price model. We also assume that this is the case in the sticky wage model (which requires that the AD curve be sufficiently steep). In contrast, in either version of the Keynesian model output will react more to demand shocks than in the neoclassical model. A key difference between the two variants of the Keynesian model is the behavior of the real
wage conditional on demand shocks. In the sticky wage model, output rises with positive demand shocks because of a falling real wage – the price level rises when the AD shifts out, which causes the real wage to decline given a fixed nominal wage, and induces firms to hire more labor. In the sticky price model, a rising price level due to an outward shift of the AD curve means that some firms have relative prices that are suboptimally low, which induces them to produce more than they otherwise might like. To produce this extra output they have to offer workers a higher real wage. So the real wage is procyclical conditional on a positive demand shock in the sticky price Keynesian model and countercyclical conditional on a demand shock in the sticky wage model.

As noted above, think about either version of the Keynesian model as holding only in the short run. How do we transition from the short run to the medium run? In either version of the Keynesian model, if \( Y_t \neq Y_t^f \), then there will be upward or downward pressure on \( W \) (in the sticky wage case) or \( P_t^e \) (in the sticky price case) to adjust. This means that a situation in which \( Y_t \neq Y_t^f \) will be followed by the AS curve shifting to “close the gap” as the economy transitions from short run to medium run. This will be true in either version of the Keynesian model:

This implies that there ought to exist a position relationship between the output gap, \( Y_t - Y_t^f \), and changes in the price level (or inflation). Positive output gaps put upward pressure on prices and negative output gaps put downward pressure on prices. This sort of relationship is often called a Phillips Curve, and is somewhat supported by the data.

3. Monetary Policy:

(a) We have heretofore thought of \( M_t \) as exogenous. In the real world, central banks adjust the money supply (and hence also interest rates) in response to economic conditions. How do they do that? How ought they to do that?

(b) Either version of the Keynesian model is just a special case of the neoclassical model. We saw earlier that the equilibrium of the neoclassical model is efficient in the sense of the allocations emerging in that equilibrium are the same allocations that a fictitious social
planner would choose so as to maximize the lifetime utility of households. Denoting $Y_t^f$ as the neoclassical equilibrium level of output, then the desired level of output in either version of the Keynesian model is $Y_t = Y_t^f$.

(c) Hence, in response to shocks, optimal monetary policy involves the adjustment of $M_t$ (and hence interest rates) so as to support $Y_t^f$ as the equilibrium of the Keynesian model. This is true in either the sticky price or sticky wage variants of that model.

(d) Since output “overreacts” to demand shocks (relative to the neoclassical model) in the Keynesian model, optimal monetary policy involves counteracting demand shocks with $M_t$. Hence, if there is a shock which shifts the AD curve to the right, monetary policy will want to reduce the money supply (and hence raise interest rates) so as to shift the AD curve back in. This is true in either version of the model. We sometimes call this contractionary or countercyclical monetary policy – you move $M_t$ in the opposite direction of how $Y_t$ would move in response to a shock.

(e) Since output “underreacts” to supply shocks (relative to the neoclassical model) in the Keynesian model, optimal monetary policy involves increasing the money supply (and therefore lowering interest rates) in response to positive supply shocks. This is true in both variants of the Keynesian model. This is sometimes called accommodative or procyclical policy, in the sense of moving $M_t$ in the same direction of how $Y_t$ would react to a shock.

(f) The following table summarizes how $M_t$ (as well as $r_t$ and $i_t$) ought to be adjusted in response to different kinds of shocks in either version of the Keynesian model:

<table>
<thead>
<tr>
<th>$M_t$</th>
<th>$r_t$ &amp; $i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(g) While the desired qualitative movements in $M_t$ in response to shocks are the same in both versions of the Keynesian model, there is an important but subtle difference in terms of the desirability of price stabilization. Conditional on demand shocks, in either version of the Keynesian model it is optimal to adjust $M_t$ in such a way that $P_t$ does not react – i.e. price stability is a good goal. This is also true conditional on supply shocks in the sticky price model. This is not true conditional on supply shocks in the sticky wage model. To implement the neoclassical equilibrium (which you can think of as occurring where the labor demand and supply curves would intersect), the real wage must react to supply shocks. With a fixed nominal wage, the only way for the real wage to react to these shocks is for the price level to change. Hence, price stability is not desirable conditional on supply shocks in the sticky wage model.

(h) Because $Y_t^f$ is not necessarily observed, but is rather a hypothetical model-based construct, the optimal monetary policy of simply adjusting $M_t$ so as to support $Y_t = Y_t^f$ as the equilibrium of the Keynesian model is easier said than done. But in the sticky price model, you don’t necessarily need to observe $Y_t^f$ to implement optimal monetary policy – you simply need to stand ready to adjust $M_t$ so as to target a constant price level (i.e. to follow a policy of price stability). That stabilizing the price level automatically stabilizes the output gap (i.e. implements $Y_t = Y_t^f$) is sometimes called the “Divine Coincidence” and is the basis of many explicit or implicit goals of central banks for
price/inflation stability. Price stability is not necessarily a good goal in the sticky wage model, at least conditional on supply shocks. In this sense monetary policy is harder in the sticky wage model than in the sticky price model.

(i) The Taylor rule is an empirical characterization of monetary policy framed in terms of a target value of the nominal interest rate:

\[ i_t = i^* + \phi_\pi (\pi_t - \pi^*) + \phi_Y (Y_t - Y_t^f) \]

The coefficients \( \phi_\pi \) and \( \phi_Y \) are assumed to be positive; Taylor argued for values of 1.5 and 0.5, respectively. The Taylor rule does a remarkably good job of capturing actual movements in interest rates over time in the data (at least until the recent period of the zero lower bound). It also fits qualitatively in with our discussion of optimal monetary policy above. Positive demand shocks raise both inflation and output relative to potential (where potential is understood to be \( Y_t^f \)). The Taylor rule calls for an increase in \( i_t \) in response to this, which involves reducing \( M_t \). Positive supply shocks lower inflation and output relative to potential (output rises after a positive supply shock absent any monetary policy accommodation, but by less than \( Y_t^f \)). The Taylor rule therefore calls for \( i_t \) to decline after a positive supply shock, which entails increasing the money supply. The Taylor rule thus seems to be both a good positive description of monetary policy (i.e. a description of how monetary policy is actually set) as well as a good normative proscription for monetary policy (i.e. its implications qualitatively align with what we discussed above).

(j) The zero lower bound refers to the fact that nominal interest rates cannot be negative. This is true whenever there is money that is storable (one of the functions which defines money). Mathematically, we must have \( i_t \geq 0 \). Since we take expected inflation to be exogenous, the lower bound on nominal interest rates imposes a lower bound on the real interest rate of \( r_t = -\pi_t^e + 1 \). The real interest rate can be negative but it cannot be more negative than the negative of expected inflation.

We can simulate the effects of the zero lower bound (ZLB) by thinking about what things would look like if policy were conducted so as to peg the nominal interest rate at a fixed value, \( i \). This would make the LM curve horizontal at \( r_t = i - \pi_t^e + 1 \). Pegging the nominal interest rate effectively pegs the real interest rate, and then output is determined by reading \( Y_t \) off of the IS curve at that real interest rate. This has the effect of making the AD curve vertical, as shown below:
(k) We can think about the ZLB as introducing a kink into the LM curve at the lower bound on \( r_t \) implied by the lower bound of 0 on the nominal interest rate. Suppose that the economy initially sits in an equilibrium where the ZLB is not binding. A sufficiently large negative shock to the IS curve could cause the ZLB to bind, which would make the AD curve vertical. This is shown below:
(l) We can think about a binding zero lower bound as exacerbating the effects of sticky prices or wages. Whereas in the neoclassical model output is solely supply-determined (because the AS curve is vertical), with a binding ZLB in the Keynesian model output is solely demand determined (because the AD curve is vertical). Demand shocks have much bigger effects with a binding ZLB than they would without a binding ZLB, and supply shocks have much smaller effects on output.

(m) The zero lower bound is problematic for two reasons. First, normal monetary policy is not available at the ZLB, and the economy will react very suboptimally to shocks (as discussed above). Second, the natural dynamics of the model will tend to make the problem of the ZLB worse. We sometimes call this a deflationary spiral. If the economy finds itself with $Y_t < Y_t^f$ at a binding ZLB, the normal dynamics of the model would tend to push the AS out to try to “close the gap.” But with a vertical AD curve, this won’t close the gap, and will only result in $P_t$ falling. Eventually, agents may begin to anticipate future falls in prices, which could manifest as lower values of expected future inflation, $\pi_{t+1}^e$. Lower expected future inflation shifts the LM curve up, and with a fixed nominal interest rate raises the real interest rate. This depresses spending along the IS curve and shifts the AD curve in to the left. A situation of a deflationary spiral is depicted graphically below:
Given that the ZLB is highly problematic, how might we escape. There are, broadly speaking, two available options. One involves non-standard monetary policy. Essentially monetary policy would want to manipulate inflation expectations to engineer and increase in $\pi_{t+1}^e$. This would shift the LM curve out, and for a fixed nominal interest rate would result in a lower real interest, thereby stimulating spending and shifting the AD curve out to the right. Higher inflation expectations could be generated via forward-guidance (essentially promising expansionary monetary policy far off into the future) or quantitative easing (which seeks to lower longer maturity interest rates). In terms of our model, we can think about these policies as trying to raise expected inflation, which graphically has sort of the inverse effects of the deflationary spiral highlighted above:
0: initial equilibrium in which ZLB binds, so real interest rate equal to negative expected inflation

Central bank convinces public of higher future inflation, $\pi_{t+1}

1: this reduces real interest rate, shifts LM down/right so that output increases and ZLB no longer binds; AD goes back to downward-sloping

One could also entertain escaping the ZLB with fiscal policy (increases in $G_t$, decreases in $G_{t+1}$, or changes in taxes to the extent to which Ricardian Equivalence does not hold). Because the real interest rate is fixed at the ZLB, there is no “crowding out” associated with fiscal expansion, and fiscal expansion ought to therefore be relatively more potent at stimulating output than in normal times.
0: initial equilibrium in which ZLB binds, so real interest rate equal to negative expected inflation

Fiscal policy increases government spending (or decreases taxes if no Ricardian equivalence)

1: this shifts the IS curve to the right so that the ZLB no longer binds

(o) One option for avoiding the ZLB in the first place is to raise the long run inflation target, which can be accomplished by increasing the growth rate of money relative to output. A higher average value of inflation means that the average value of the nominal interest rate ought to be higher, which ought to reduce the frequency of running into the zero constraint. The downside of doing this is that there may be costs of high inflation in the long run which our model does not fully capture. Balancing these benefits of a higher inflation target (reduced incidence of hitting the ZLB) with the costs quantitatively seems to suggest an optimal long run inflation target of about 2 percent, which is roughly what the central bank in the US and many others around the world seem to follow.

4. Great Recession

(a) The Great Recession, officially date from December of 2007 to June of 2009, represents the largest economic contraction in the US (and most of the rest of the world) since the
Great Depression. Output fell by about 10 percent relative to trend, the unemployment rate doubled, and the recovery has been weak, with output failing to catch back up to trend.

(b) It is useful to divide the Great Recession into three phases as follows:

i. Stage 1: house prices, which had expanded enormously in the first part of the decade, slowed down in 2006 and began to collapse into 2007. Housing wealth, $HW_t$, is a form of wealth for households, and could be a component of consumption: $C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t, HW_t)$. The decline in housing wealth in 2007 and continuing into 2008 would have caused households to desire less consumption. This would result in an inward shift of the IS and hence AD curves. The Fed optimally responded to this with expansionary monetary policy by increasing the money supply and cutting interest rates. The end result was not much change in output by the middle part of 2008, but interest rates were very low and the ZLB was effectively binding by that time. Graphically, we have:

Reduction in $HW$ (housing wealth) shifts IS and hence AD curve, resulting in falling real interest rate, output, and price level.
(c) Stage 2: the financial crisis. Largely as a consequence of the housing market collapse, the financial system collapsed at the end of 2008 and into early 2009. This occurred because of heightened exposure of large financial institutions to the housing market through things like mortgage backed securities, as well as increasing interconnectedness of the financial system due to exotic insurance products. The financial crisis manifested itself in terms of a collapse in stock prices and heightened credit spreads. In terms of our model, we can think about this as being represented by a decline in $q$. By the time the financial system collapsed, the zero lower bound was effectively binding. This meant that the output reduction associated with the reduction in $q$ was much larger than it would have been away from the zero lower bound, and also meant that the normal monetary policy of combatting a decline in $q$ with an increase in the money supply and a cut in interest rates was not available. Graphically:

Reduction in $HW$ (housing wealth) shifts IS and hence AD curve, resulting in falling real interest rate, output, and price level. Fed responds by increasing the money supply and lowering interest rates. Little change in output or prices by end of 2007, but interest rates are low.
In the data, output declined most drastically at the end of 2008 and in the first half of 2009. This corresponds with the implications in the model of a large negative financial shock occurring towards the end of 2008. Inflation also fell, consistent with a negative aggregate demand shock.

(d) Stage 3: policy responses. The policy responses to the financial crisis were unprecedented. We can essentially divide these into three components:

i. Financial market interventions: through things like TARP and bailouts, the Fed and US Treasury tried to directly intervene in financial markets, serving as a sort of lender of last resort to try to ensure that financial institutions remained solved and that credit markets didn’t completely dry up. We can think about this mechanically in the model as policymakers directly trying to undo the reduction in $q$.

ii. Non-Standard monetary policy: through forward-guidance and quantitative easing, the Fed tried to engineer higher expected inflation by promising extend periods of expansionary monetary policy far off into the future. In terms of our model, we can think about these efforts as trying to raise $\pi^e_{t+1}$, which would have the effect of lowering the real interest rate, stimulating spending, and shifting the AD curve to the right.

iii. Fiscal policy: the American Recovery and Reinvestment Act was a large stimulus.
package spread out over several years composed of both spending increases and tax
cuts. The objective was to prop up demand by shifting the IS curve to the right. With a binding ZLB, fiscal policy is expected to be particularly potent since there is no crowding out.

(e) It is difficult to definitively say whether the extreme policy measures after the Great Recession worked or not, as we do not know what would have happened in the absence of these policy interventions. We did not have a repeat of the Great Depression, and the economy began growing again in the latter half of 2009. That said, the recovery has been weak, and policy remains quite loose. Some have argued that the extraordinary policy measures have led to heightened uncertainty which has partially depressed demand and served to hold back the recovery some.