In this document we explore issues related to fiscal policy – that is, government spending, taxes, and debt. In a conventional, flexible price, market-clearing model, one can show that there is no welfare motivation for government to consume any output. As such, in this document and in this class I take government expenditure as exogenously given. You could make it optimal for government spending to be non-zero in a couple of ways. First, you could put it directly in the utility function of the representative agent (i.e. kind of like money in the utility function). There are different ways you could do this – making it additively separable or making there be a positive cross partial with consumption and/or leisure. A better way to is to assume that there are public goods which would be under-provided by private agents, and so government can increase welfare by providing the good.

**Ricardian Equivalence**

We begin with the celebrated Ricardian Equivalence result. Essentially, Ricardian equivalence says that households are indifferent between tax and debt finance of government purchases (provided the taxes are lump sum) – that is, there is an equivalence between the two kinds of finance, and the response of the economy to a change in government spending is the same whether it is tax financed, debt financed, or some combination thereof. This has several important implications. The first is that households do not care about the time path of taxes if those taxes are lump sum – what they care about is the time path of government spending. Thus a tax cut without a change in spending has no effect on the household. Secondly, deficit financed increases in spending do not have larger effects on economic activity than do tax financed increases in spending, which was once the received wisdom from the Keynesian literature.

To see the point concretely, examine the household problem. I assume that households own the capital stock and receive rental rate $r_t^k$ on their holdings of capital each period:

$$\max_{c_t, k_t, b_t, l_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) + v(l_t))$$

s.t.

$$n_t + l_t = 1$$
$$c_t + k_{t+1} + b_{t+1} \leq w_t n_t + (1 + r_t^k) k_t - T_t + (1 + r_t^b) b_t$$

Bonds, $b_t$, are issued by the government. The government chooses its time path of spending exogenously, and it must satisfy its budget constraint:

$$g_t - b_{t+1} \leq T_t - (1 + r_t^b) b_t$$
That debt enters with a negative sign because the government can increase its current spending without raising taxes by raising \( b_{t+1} \), which is an asset for the household. Assume that the government budget constraint holds with equality, and solve for next period’s bondholdings:

\[
b_{t+1} = -T_t + g_t - (1 + r_t^b) b_t
\]

Plug this into the household budget constraint:

\[
c_t + k_{t+1} - T_t + g_t - (1 + r_t^b) b_t \leq w_t n_t + (1 + r_t^k) k_t - T_t + (1 + r_t^b) b_t
\]

Simplify:

\[
c_t + k_{t+1} \leq w_t n_t + (1 + r_t^k) k_t - g_t
\]

In other words, one way to think about Ricardian equivalence is that households behave as though the government balances its budget period by period (i.e. \( g_t = T_t \)). If spending goes up, people will feel poorer even if taxes don’t change immediately. Likewise, if taxes go down but spending doesn’t change (and isn’t expected to change), then people won’t feel any poorer or richer than they did before – in other words the tax cut has no effect.

We can attempt to say something about the time path of taxes given an exogenous time path of spending by solving the government budget constraint forward to find the relationship between current debt and future spending and taxes:

\[
g_t - b_{t+1} = T_t - (1 + r_t^b) b_t
\]

\[
b_t = \frac{T_t - g_t}{(1 + r_t^b)} + \frac{b_{t+1}}{(1 + r_t^b)}
\]

Solving forward and using the no ponzi-condition that the present discount value of future debt limit to zero:

\[
b_t = \frac{T_t - g_t}{(1 + r_t^b)} + \frac{1}{(1 + r_t^b)} \left( \frac{T_{t+1} - g_{t+1}}{(1 + r_t^b)} + \frac{b_{t+2}}{(1 + r_t^b)} \right)
\]

\[
b_0 = \sum_{t=0}^{\infty} \prod_{j=0}^{t} (1 + r_j)^{-1} (T_t - g_t)
\]

In other words, current debt must be equal to the present discounted value of future surpluses (revenue less expenditure). This can be re-arranged to yield:

\[
b_0 + \sum_{t=0}^{\infty} \prod_{j=0}^{t} (1 + r_j)^{-1} g_t = \sum_{t=0}^{\infty} \prod_{j=0}^{t} (1 + r_j)^{-1} T_t
\]

In other words, the present discounted value of lump sum taxes must equal initial debt plus the present discounted value of spending.
A change in taxes that is not met with a change in spending simply means that taxes must change in the opposite direction (and in an equal magnitude in present value) at some point in the future. An increase in spending that is not met by an immediate change in taxes means that taxes will go up at some point in the future (and in an equal magnitude in present value). Since households are forward-looking, all they care about is the present discounted value of their obligations to the government. Since debt and tax financed increases in spending must have the same effect on the present value of obligations, these are equivalent from the perspective of households.

Ricardian equivalence is a striking result that is very much counter to Keynesian intuition. That being said, it is not very generalizable. First, it requires that people live forever, which, while a convenient short cut, is obviously counterfactual. It also requires that people be forward-looking, not have hyperbolic discounting, and that there be no credit constraints. Finally, it also requires that taxes be non-distortionary (i.e. lump sum). This is perhaps the most counterfactual of all assumptions. Does this mean that the Ricardian equivalence result is useless? No! It provides important insights about reasons why fiscal policy isn’t likely to be as effective as is assumed in most Keynesian type models.

**Distortionary Taxation**

We now add distortionary taxation to the problem. We do this in a simple way – households pay proportions of their labor and capital income in taxes, as well as a tax on consumption expenditures. We also think about the government behaving optimally. The equivalence between the planner’s and decentralized problem breaks down here, so we solve it in steps. The household solves his/her problem taking the government tax rates as given in the first stage. In the second stage the government picks tax rates to maximize household welfare given the optimal decision rules from the first stage. We take the time path of spending as given and do not allow the government access to lump sum taxes. This is necessary in order to make the problem interesting – a benevolent government would not (i) not do any spending (absent an explicit public good) and (ii) if it had to spend, would finance it with lump sum taxes if it could.

Households face the following problem:

$$\max_{c_t, k_t, b_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) + v(l_t))$$

s.t.

$$n_t + l_t = 1$$

$$(1 + \tau^c_t) c_t + k_{t+1} + b_{t+1} \leq (1 - \tau^w_t) w_t n_t + (1 + (1 - \tau^k_t) r^k_t) k_t + (1 + r^k_t) b_t$$

Here $r^k_t$ is equal to the marginal product of capital less depreciation. The Lagrangian for the problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + v(1 - n_t) + \lambda_t \left( (1 - \tau^w_t) w_t n_t + (1 + (1 - \tau^k_t) r^k_t) k_t + \ldots + (1 + r^k_t) b_t - (1 + \tau^k_t) c_t - k_{t+1} - b_t \right) \right\}$$
The first order conditions are:

\[
\begin{align*}
\frac{\partial L}{\partial c_t} &= 0 \iff u'(c_t) = \lambda_t (1 + \tau^r_t) \\
\frac{\partial L}{\partial n_t} &= 0 \iff v'(1 - n_t) = \lambda_t (1 - \tau^w_t) w_t \\
\frac{\partial L}{\partial b_{t+1}} &= 0 \iff \lambda_t = \beta E_t (1 + r^b_t) \lambda_{t+1} \\
\frac{\partial L}{\partial k_{t+1}} &= 0 \iff \lambda_t = \beta E_t (1 + (1 - \tau^k_{t+1}) r^k_{t+1}) \lambda_{t+1}
\end{align*}
\]

Combining the last two conditions yields a “no arbitrage” condition on bonds and capital. In particular, it must be the case that:

\[(1 + r^b_{t+1}) = (1 + (1 - \tau^k_{t+1}) r^k_{t+1})\]

This condition essentially requires that the household be indifferent, in equilibrium, to saving through bonds or capital. It is clear that if the tax rate on capital is zero, then these two rates of return must be equal. Substituting the other first order conditions together yields:

\[
\begin{align*}
u'(c_t) &= \beta E_t (1 + r^b_{t+1}) \left( \frac{1 + \tau^r_t}{1 + \tau^k_{t+1}} \right) u'(c_{t+1}) \\
v'(1 - n_t) &= u'(c_t) \left( \frac{1 - \tau^w_t}{1 + \tau^r_t} \right) w_t
\end{align*}
\]

Firms are competitive. They rent capital and labor from households each period in competitive spot markets. The firm problem is:

\[
\max_{k_t, n_t} f(k_t, n_t) - w_t n_t - (r^k_t + \delta) k_t
\]

The function \(f(\cdot)\) is homogenous of degree one, increasing and concave in both arguments, and with a positive cross-partial. The first order conditions are:

\[
\begin{align*}
r^k_t &= f_1 (k_t, n_t) - \delta \\
w_t &= f_2 (k_t, n_t)
\end{align*}
\]

The government budget constraint is:

\[
\tau^w_t w_t n_t + \tau^k_t r^k_t k_t + \tau^r_t c_t - (1 + r^b_t) b_t \geq g_t - b_{t+1}
\]

The government’s objective is to maximize household utility subject to (i) satisfying its budget constraint, (ii) households and firms optimizing, and (iii) the aggregate resource constraint holding. This means that the government takes the first order conditions of
households and firms as given. We’re going to include the first order conditions for the households as constraints in the problem. We will impose the firm conditions by including them in the government budget constraint.

Since the production function is homogeneous of degree one, we can write it as the sum of payments to the two factors:

\[ f(k_t, n_t) = w_t n_t + (r^k_t + \delta)k_t \]

We can then write:

\[ \tau^w_t w_t n_t + \tau^k_t r^k_t k_t = f(k_t, n_t) - w_t n_t - (r^k_t + \delta)k_t + \tau^w_t w_t n_t + \tau^k_t r^k_t k_t \]

\[ \tau^w_t w_t n_t + \tau^k_t r^k_t k_t = f(k_t, n_t) - (1 - \tau^w_t)w_t n_t - (1 - \tau^k_t)r^k_t k_t - \delta k_t \]

This way we’ve imposed the firm first order conditions into the government budget constraint:

\[ f(k_t, n_t) - (1 - \tau^w_t)w_t n_t - (1 - \tau^k_t)r^k_t k_t - \delta k_t + \tau^c_t c_t - (1 + r^b_t)b_t \geq g_t - b_{t+1} \]

The Lagrangian for the government is:

\[
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + v(1 - n_t) + \ldots + \mu_t (f(k_t, n_t) - (1 - \tau^w_t)w_t n_t - (1 - \tau^k_t)r^k_t k_t - \delta k_t + \tau^c_t c_t - (1 + r^b_t)b_t - g_t + b_{t+1}) + \ldots + \gamma_{1,t} \left( (1 + (1 - \tau^k_{t+1})r^k_{t+1}) - (1 + r^b_{t+1}) \right) + \gamma_{2,t} \left( u'(c_t) \left( \frac{1+r^b_{t+1}}{1+r^b_t} \right) w_t - v'(1 - n_t) \right) + \ldots + \gamma_{3,t} \left( \beta E_t + r^b_{t+1} \left( \frac{1+r^b_{t+1}}{1+r^b_t} \right) u'(c_{t+1}) - u'(c_t) \right) + \ldots + \gamma_{4,t} \left( f(k_t, n_t) - c_t - g_t - k_{t+1} + (1 - \delta)k_t \right) \right\}
\]

The first constraint is the government budget constraint (combined with the firm FOC). The second, third, and fourth are the household FOC. The final constraint is the aggregate resource constraint. The first order conditions for the government’s problem are:
\[
\frac{\partial L}{\partial t_{+1}} = 0 \iff \mu_t = \beta E_t \mu_{t+1} (1 + r^b_{t+1})
\]

\[
\frac{\partial L}{\partial c_t} = 0 \iff u'(c_t) + \mu_t \tau^c_t + \gamma_{2,t} u''(c_t) \left( \frac{1 - \tau^w_t}{(1 + \tau^c_t)} \right) w_t + \gamma_{3,t-1} (1 + r^b_t) \left( \frac{1 + \tau^c_{t-1}}{1 + \tau^c_t} \right) u''(c_t) = \gamma_{3,t} u''(c_t) + \gamma_{4,t}
\]

\[
\frac{\partial L}{\partial k_{t+1}} = 0 \iff \beta E_t \mu_{t+1} \left( f_1(k_{t+1}, n_{t+1}) - (1 - \tau^k_{t+1}) r^k_{t+1} - \delta \right) + \beta E_t \gamma_{4,t+1} \left( f_1(k_{t+1}, n_{t+1}) + (1 - \delta) \right) = \gamma_{4,t}
\]

\[
\frac{\partial L}{\partial n_t} = 0 \iff \mu_t \left( f_2(k_t, n_t) - (1 - \tau^w_t) w_t \right) + \gamma_{4,t} f_2(k_t, n_t) = v'(1 - n_t) - \gamma_{2,t} v''(1 - n_t)
\]

\[
\frac{\partial L}{\partial \tau^k_t} = 0 \iff \mu_t r^k_t k_t = \frac{1}{\beta} \gamma_{1,t-1} r^k_t
\]

\[
\frac{\partial L}{\partial \tau^w_t} = 0 \iff \mu_t w_t n_t = \gamma_{2,t} u'(c_t) \left( \frac{w_t}{(1 + \tau^w_t)} \right)
\]

\[
\frac{\partial L}{\partial \tau^c_t} = 0 \iff \mu_t c_t + \gamma_{3,t} \left( \beta E_t (1 + r^b_{t+1}) \left( \frac{u'(c_{t+1})}{(1 + \tau^c_{t+1})} \right) \right) = \gamma_{2,t} u'(c_t) \left( \frac{1 - \tau^w_t}{(1 + \tau^c_t)^2} \right) w_t + \gamma_{3,t-1} \left( (1 + r^b_t) \frac{1 + \tau^c_{t-1}}{1 + \tau^c_t} \right) u'(c_t)
\]

These equations are (obviously) not that easy to make sense of. There is, however, one important result that can be derived. Take the first order condition for the choice of the future capital stock, and evaluate it at the non-stochastic steady state:

\[
\beta \mu^* \left( f_1(k^*, n^*) - (1 - \tau^{k*}) r^{k*} - \delta \right) + \beta \gamma_{4}^* \left( f_1(k^*, n^*) + (1 - \delta) \right) = \gamma_{4}^*
\]

We can simplify this by noting that:

\[
f_1(k^*, n^*) = r^{k*} + \delta
\]

Then we have:

\[
\beta \mu^* \left( \tau^{k*} r^{k*} \right) + \beta \gamma_{4}^* \left( 1 + r^{k*} \right) = \gamma_{4}^*
\]

From the household first order condition (which must hold), we know that:

\[
(1 + (1 - \tau^{k*}) r^{k*}) = \frac{1}{\beta}
\]

We can write this as:

\[
1 = \beta \left( 1 + (1 - \tau^{k*}) r^{k*} \right)
\]

\[
1 = \beta(1 + r^{k*}) - \beta \tau^{k*} r^{k*}
\]

\[
\beta(1 + r^{k*}) = 1 + \beta \tau^{k*} r^{k*}
\]

Plugging this into the government’s first order conditions, we see that:
\[ \beta \mu^* (r^{k*} r^{k*}) + \gamma_4^* (1 + \beta r^{k*} r^{k*}) = \gamma_4^* \]

Simplifying:

\[ \beta \mu^* (r^{k*} r^{k*}) + \gamma_4^* + \beta \gamma_4^* r^{k*} r^{k*} = \gamma_4^* \]

\[ \beta r^{k*} r^{k*} (\mu^* + \gamma_4^*) = 0 \]

The only way that this can hold is if \( r^{k*} = 0 \). We thus have the celebrated result from Chamley (1986) and Judd (1985) that the optimal rate of capital taxation in the long run is zero.

What is the intuition for this result? A key finding in the optimal taxation literature in public finance is that the optimal tax rate on a commodity is related to its supply elasticity. Goods with relatively inelastic supply should face relatively high tax rates; goods with elastic supply should face low tax rates. The intuition for this finding is relatively straightforward. If supply is elastic, then agents will be able to change their behavior by a lot to different tax rates; if supply is relatively inelastic, then they can’t change their behavior much, so the tax doesn’t lead to much of a distortion. Since a benevolent government would want to maximize efficiency/minimize distortions, it will tax most heavily those goods for which the resulting distortions from taxation are lowest.

Let’s think about demand and supply of capital. The demand for capital is given by the firm’s first order condition:

\[ r^k_t = f_1 (k_t, n_t) - \delta \]

Given the concavity of the production function, this is downward sloping. Let’s think about the supply of capital. Capital is supplied by households when they save; they are being taxed on their holdings of capital, so the capital tax drives a wedge between what they get for owning capital and what the firm pays. In the short run, the supply of capital is completely inelastic – it is predetermined with respect to current taxes and anything else. Thus, it is optimal for the government to tax the heck out of holders of capital in the first period – there is nothing they can do to avoid it, and there is thus no efficiency loss. But in the long run, the supply of capital is effectively perfectly elastic, determined by the discount factor. Taxing something supplied perfectly elastically leads to large efficiency losses.

These points can be made graphically with simple supply and demand graphs. For some arbitrary commodity, we can depict equilibrium graphically. The area under the demand curve and above the price is consumer surplus; the area above the supply curve and under the price is producer surplus. Total surplus is the sum of these two areas.
A tax (assume it is paid by the supplier), drives a wedge between the price paid by the demand side and the price received by the suppliers:

Total surplus is now equal to consumer surplus, plus producer surplus, plus tax revenue. But total surplus with the tax is now less than it was before; the amount by which it is less is given by the triangle labeled deadweight loss. It is straightforward to see that, as the
supply curve becomes vertical (which is what the capital supply curve is in the very short run), then the deadweight loss is equal to zero – tax revenue just replaces producer surplus, with no change in consumer surplus. But if supply is very elastic (i.e. horizontal or near horizontal), then the deadweight/efficiency loss is large. Since the effective supply of capital is perfectly horizontal in the long run, it’s optimal to not tax it in the long run.

This raises an interesting issue as pertains optimal government finance. Taxing capital today is like a lump sum tax and hence results in no efficiency loss. But taxing capital in the long run results in a big efficiency loss. So what the government would like to do is to institute capital levies today to finance expenditure and commit to lowering the tax rate on capital in the long run. But this isn’t credible – people will know that the government will face the same choice in the future. In other words, there is a time inconsistency here – the government will solve its problem differently at each date than it had planned to solve the problem previously.