Graduate Macro Theory II: Fiscal Policy in the RBC Model

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1 Introduction

This set of notes studies fiscal policy in the RBC model. Fiscal policy refers to government spending and finance. Government spending is a component of aggregate expenditure; we could model it as potentially solving some public goods provision problem or as being productive (by productive I mean that government spending helps firms produce output one way or another). We will do neither and assume that government spending is pure consumption for the government. We all for the possibility that the representation household gets utility from government spending, which is a reduced-form way to model the "usefulness" of government spending (which is to provide public goods that would be under-provided by private markets).

In our setup the government will choose spending exogenously; we could also model a Ramsey type problem where spending is chosen to maximize social welfare. The government has a flow budget constraint that must hold and finances itself with some mixture of debt and taxes; in the long run, debt cannot explode, so debt today necessitates raise tax revenue at some point in the future. We consider both "lump sum" taxes (the amount of tax an agent pays is independent of any choices it makes), as well as "distortionary" taxes which are tax rates on income or spending. We call these "distortionary" because they distorted inter or intratemporal first order conditions that would emerge in the solution to a planner's problem when the planner has access to lump sum taxes.

2 Lump Sum Finance

We begin by assuming that all government revenue is raised through lump sum taxes. While unrealistic, this is a good starting point. Among other things it means that the competitive equilibrium can be supported as the solution to a social planner's problem where the amount of spending is exogenously given. We will analyze the effects of changes in government spending and talk about an important concept called Ricardian Equivalence, which says that, if taxes are lump sum, the mix between debt and tax finance is irrelevant.

Below I set up the problems of the different agents and then discuss equilibrium. I assume that the household owns the capital stock and leases it to firms.

2.1 Household

There is a representative household that solves a standard problem, owning capital and leasing it to firms. Its problem is:

$$\max_{C_t, N_t, K_{t+1}, B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} + h(G_t) \right)$$

s.t.

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} - B_t \leq w_t N_t + R_t K_t + \Pi_t - T_t + r_{t-1} B_t$$

The household can consume and supplies labor; it can save through capital or bonds. It receives profit from firms, Π_t, and potentially pays taxes to the government, T_t. It takes both of these as given. We call T_t a "lump sum" tax because it the amount the household pays is independent of anything the household does – it pays T_t whether it works a lot or a little, etc. In the setup of the problem I suppose that the household may get some utility from government spending, G_t, through $h(G_t)$. The household takes G_t as given. As long as the utility from government spending is additively separable in the utility function, it will not affect household choices or the equilibrium dynamics. I put it there simply as a reduced form way to justify why the government might choose to do any government spending in the first place.

The first order conditions for the household problem are standard:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(R_{t+1} + (1-\delta) \right) \right] \tag{1}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1 + r_t) \right] \tag{2}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} w_t \tag{3}$$

The above are the FOC for capital, bonds, and labor, and are entirely standard.

2.2 Firm

There is a representation firm that solves the standard static firm problem:

$$\max_{N_t, K_t} \quad \Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t K_t$$

The FOC are to equate factor prices with marginal products, again entirely standard:

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{4}$$

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{5}$$

2.3 Government

The government chooses spending, G_t, exogenously. It finances spending with the aforementioned lump sum taxes, T_t, and by issuing new debt, D_{t+1}. The government inherits an existing stock of debt, D_t, from history. We will impose in the equilibrium that any debt issues by the government is held by the household. We do not model the government's problem, and as we will see below the exact mix between taxes and debt is both indeterminate and irrelevant for the equilibrium dynamics. All that we require is that the government's debt not explode.

The government's flow budget constraint is:

$$G_t + r_{t-1} D_t \leq T_t + D_{t+1} - D_t \tag{6}$$

Government spending plus interest payments on existing debt cannot exceed tax revenue plus new debt issuance. So if G_t increases in a period, there are two ways to pay for it – increase current taxes, T_t, or issue more debt, $D_{t+1} - D_t$.

2.4 Equilibrium

A competitive equilibrium is a set of prices (r_t, R_t, w_t) and allocations $(C_t, K_{t+1}, N_t, B_{t+1}, D_{t+1})$ such that (i) household and firm optimality conditions all hold, (ii) the firm hires all the labor and capital supplied by the household, (iii) the household and firm budget constraints hold with equality, and (iv) household bond-holdings equal government debt issuance in all periods (e.g. $B_{t+1} = D_{t+1}$), given values and stochastic processes of G_t and A_t, as well as initial values of government debt and household bond-holdings, which must be equal (e.g. $B_t = D_t$).

The government budget constraint binding with equality means that:

$$T_t = G_t + (1 + r_{t-1}) D_t - D_{t+1}$$

Plug this, along with the definition of profit, into the household budget constraint at equality:

$$C_t + K_{t+1} - (1-\delta) K_t = A_t K_t^{\alpha} N_t^{1-\alpha} - G_t - (1 + r_{t-1}) D_t + D_{t+1} + (1 + r_{t-1}) B_t - B_{t+1}$$

Using the fact that $D_t = B_t$ and $D_{t+1} = B_{t+1}$, we have:

$$C_t + K_{t+1} - (1-\delta) K_t + G_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{7}$$

Defining $I_t = K_{t+1} - (1-\delta) K_t$ and $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$, we can summarize the equilibrium as:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(R_{t+1} + (1-\delta) \right) \right] \tag{8}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1 + r_t) \right] \tag{9}$$

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{10}$$

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{11}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} w_t \tag{12}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{13}$$

$$Y_t = C_t + I_t + G_t \tag{14}$$

$$K_{t+1} = I_t + (1-\delta) K_t \tag{15}$$

I close the model with exogenous stochastic processes for government spending and productivity:

$$\ln G_t = (1-\rho_g) \ln (\omega Y) + \rho_g \ln G_{t-1} + \epsilon_{g,t} \tag{16}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_{a,t} \tag{17}$$

I assume that the non-stochastic steady state value of government spending is a fraction, ω, of steady state output, Y (I denote steady state values by the absence of a time subscript). We take ω to be an exogenous parameter. Productivity follows a mean zero AR(1) in the log, so that the non-stochastic steady state value of the level of A is 1. This constitutes a system of 10 variables (C_t, N_t, I_t, Y_t, w_t, R_t, r_t, K_t, G_t, and A_t) and 10 equations.

Note that in writing these equilibrium conditions debt and taxes show up nowhere. I could add the government budget constraint to the equilibrium conditions, but that would be adding one equation with two variables (T_t and D_{t+1}). What we have here is that the mix between debt and taxes is indeterminate, and, indeed, irrelevant to the equilibrium dynamics. We could assume that $T_t = G_t$ each period and we'd get the same equilibrium dynamics. All that matters is the time path of G_t, not how it is financed. This is the essential gist of Ricardian Equivalence, which we discuss next.

2.5 Diversion: Ricardian Equivalence

The fact that neither debt nor taxes show up anywhere in the equilibrium conditions means that the mix between debt and taxes is both (i) indeterminate and (ii) irrelevant. Imposing government budget balance each period, e.g. $G_t = T_t$, would yield exactly the same equilibrium dynamics. This result is called *Ricardian Equivalence*. Ricardian Equivalence states that the manner of government finance (the mix between taxes and bonds) is irrelevant for understanding the equilibrium effects of changes in government spending. It does not say that changes in government spending have no economic effects (as we will see in a minute, they do), it simply says that the manner in which they are financed is irrelevant to the equilibrium behavior in the model. This requires that taxes are lump sum and that households are forward-looking and are not subject to any liquidity constraints.

There are a couple of ways to see the intuition for what is at play here. First, assume that the government budget constraint binds with equality. This means that $T_t = G_t + (1 + r_{t-1})D_t - D_{t+1}$. Now plug this into the household's budget constraint:

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} - (1+r_{t-1})B_t = w_t N_t + R_t K_t + \Pi_t - G_t - (1+r_{t-1})D_t + D_{t+1}$$

Since the household knows that $D_t = B_t$ in equilibrium, these all drop out, leaving:

$$C_t + K_{t+1} - (1-\delta)K_t = w_t N_t + R_t K_t + \Pi_t - G_t$$

This budget constraint is identical to assuming that $B_t = D_t = 0$ (and $B_{t+1} = D_{t+1} = 0$) and that $G_t = T_t$. In other words, the household behaves as though $G_t = T_t$, whether this is in fact the case or not. This has the implication that an increase in G_t will affect the household's decision-making as a negative shock to wealth – when G_t goes up, the household will feel like it is paying more in taxes (again, whether it is in fact or not). This will prove an important insight into understanding the economic effects of changes in government spending below.

Another way to see this is to derive the *intertemporal* budget constraints for both the government and household combined with the TV/no-ponzi conditions. Start with the government budget constraint, and assume it holds with equality each period. For ease of exposition, suppose that the interest rate is constant each period with $r_{t+j} = r$ for all $j \geq 0$. This is not necessary for what follows but allows us to avoid product operators.

$$G_t + (1+r)D_t = T_t + D_{t+1}$$

This must hold in the next period as well:

$$G_{t+1} + (1+r)D_{t+1} = T_{t+1} + D_{t+2}$$

This means that:

$$D_{t+1} = \frac{1}{1+r}\left(T_{t+1} - G_{t+1} + D_{t+2}\right)$$

We can do the same thing for D_{t+2}:

$$D_{t+2} = \frac{1}{1+r}\left(T_{t+2} - G_{t+2} + D_{t+3}\right)$$

And so on. Putting all these into the period t constraint, you have:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j G_{t+j} + (1+r)D_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j T_{t+j} + \lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T D_{t+T+1}$$

A non-explosive path for government debt requires that the last term goes to zero. So the government budget constraint says that the present discounted value of spending (plus one plus the interest rate times the initial stock of debt) must equal the present discounted value of taxes. For simplicity, assume that in period t there is no debt (again, this doesn't matter, as long as the debt is held by the household). Therefore, the government's budget constraint can be written:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j G_{t+j} = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j T_{t+j} \tag{18}$$

In words, this says that the government must balance its budget in a present value sense – the present value of spending must equal the present discounted value of taxes, though the government need not balance the budget period-by-period. Now let's do something similar for the household. Using the definitions of investment and profit, we can write the household budget constraint as:

$$C_t + I_t + B_{t+1} - (1+r)B_t = Y_t - T_t$$

We can again solve this forward by recursively eliminating the savings terms:

$$B_{t+1} = \frac{1}{1+r}\left(C_{t+1} + I_{t+1} - Y_{t+1} + T_{t+1} + B_{t+2}\right)$$

If you keep doing this, you can write the household budget constraint as:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j (C_{t+j} + I_{t+j}) - (1+r)B_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j Y_{t+j} - \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j T_{t+j} + \lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T B_{t+T+1}$$

Imposing that the final term go to zero, we'd have:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j (C_{t+j} + I_{t+j}) - (1+r)B_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j Y_{t+j} - \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j T_{t+j}$$

What the household cares about on the "income" side of the budget constraint is the present discounted value of taxes. But from above we know that this equals the present discounted value of spending (here I assume that initial debt/savings is zero, but again this doesn't matter as these would cancel out), leaving:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j (C_{t+j} + I_{t+j}) = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j Y_{t+j} - \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j G_{t+j}$$

This is another way of seeing what you can see just looking at the flow budget constraint in one period: taxes drop out. All the household cares about is the present value of government spending, which must be equal to the present value of taxes. Here's a simple way to think about the intuition for this. Suppose that the government increases spending in period t; it can either tax the household now (which makes the household feel poorer) or issue debt, which the household buys as savings.

But the government debt isn't really net worth to the household: debt today simply means that taxes must be higher at some point in the future (so that the government budget balances in the present value sense), and taxes must be higher in the future by an amount exactly equal in present value to the savings that the household accumulates by buying government debt in the present. Hence, the household is indifferent to whether it pays taxes now, or buys the government debt today and pays taxes later.

2.6 The Steady State

Let's solve for the non-stochastic steady state values of the variables in the model, noting that $A = 1$ and $\frac{G}{Y} = \omega$, as well noting that we don't need to worry about T or D. I denote steady state values with the absence of a time subscript. From the Euler equation for capital, we have:

$$R = \frac{1}{\beta} - (1 - \delta) \tag{19}$$

Combining this with the first order condition for capital demand allows us to solve for the steady state capital-labor ratio:

$$\frac{K}{N} = \left(\frac{\alpha}{\frac{1}{\beta} - (1 - \delta)} \right)^{\frac{1}{1-\alpha}} \tag{20}$$

We then know that the wage satisfies:

$$w = (1 - \alpha) \left(\frac{K}{N} \right)^{\alpha} \tag{21}$$

From the production function, we then have:

$$\frac{Y}{N} = \left(\frac{K}{N} \right)^{\alpha}$$

From the accumulation equation, we know that $I = \delta K$. We can then solve for the steady state consumption-labor ratio from the resource constraint as:

$$\frac{C}{N} = (1 - \omega) \left(\frac{K}{N} \right)^{\alpha} - \delta \frac{K}{N}$$

Then go to the FOC for labor. Multiply both sides by N, we can write:

$$\theta N^{1+\chi} = \frac{N}{C} (1 - \alpha) \left(\frac{K}{N} \right)^{\alpha}$$

Solving:

$$N = \left(\frac{1}{\theta} \frac{(1 - \alpha) \left(\frac{K}{N}\right)^{\alpha}}{(1 - \omega) \left(\frac{K}{N}\right)^{\alpha} - \delta \frac{K}{N}} \right)^{\frac{1}{1+\chi}} \tag{22}$$

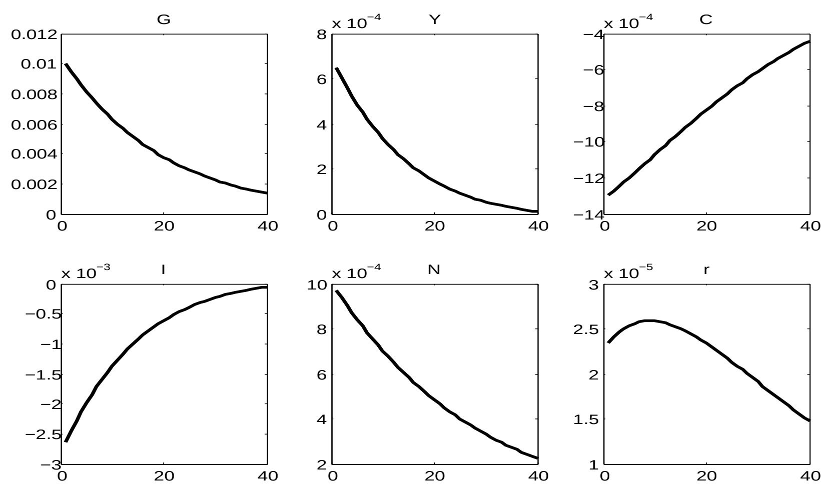
Since $\frac{K}{N}$ is independent of ω, we see here that N is increasing in ω – more government spending in steady state, more N. Since $\frac{K}{N}$ is unaffected by ω but N is increasing in ω, we see that K must be increasing in ω. Since $I = \delta K$, this means that steady state investment is higher the bigger is the government spending share. This is perhaps somewhat non-intuitive, since we often think of government spending "crowding out" private investment, but we can see this is not the case here. The intuition for what is going on is that higher ω makes people feel poorer, so they work more. More N raises the marginal product of capital, which makes it optimal to accumulate more capital, which necessitates more investment.

Since N is higher and $\frac{K}{N}$ is unaffected, we see that Y will be higher when ω is higher. Since $\frac{K}{N}$ is unaffected and N is higher, from the FOC for labor supply we see that C must be lower if ω is bigger.

2.7 Quantitative Simulations

I solve the model via log-linearizing about the non-stochastic steady state. I use the following parameter values: $\alpha = 1/3$, $\beta = 0.99$, $\chi = 1$, $\delta = 0.025$, $\theta = 4$, $\rho_a = 0.97$, $\rho_g = 0.95$, $\omega = 0.2$, and set the standard deviations of both the productivity and government spending shocks to 0.01, or 1 percent.

Below are the impulse responses to a shock to government spending:



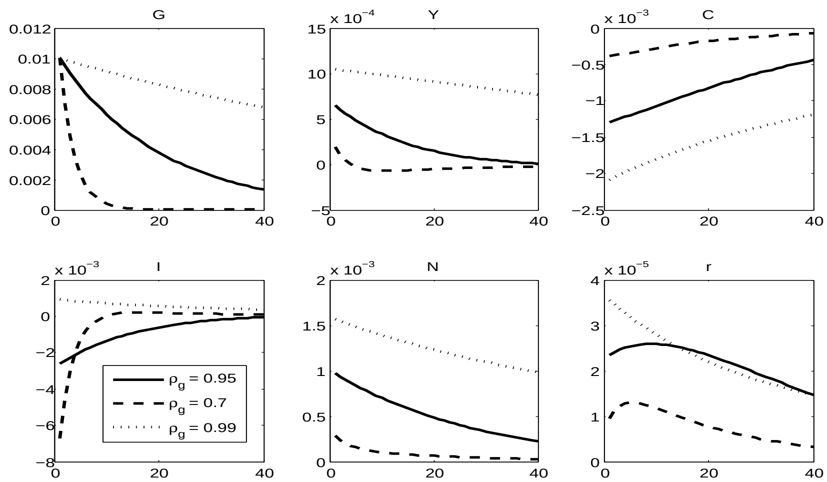
Here we see that Y_t and N_t both go up, but I_t and C_t both fall; the real interest rate rises. The government spending multiplier is often defined as $\frac{dY_t}{dG_t}$. We can calculate this from the IRFs by dividing the impact response of output by the impact response of goverment spending, and then multiplying by the inverse of the steady state government spending to output ratio (multiplication by the inverse of this ratio transforms the impact responses in the IRFs, which are percentage deviations from steady state, into actual deviations from steady state). For this setup the multiplier comes out to be 0.3237 – e.g. a one unit increase in government spending raises output by about

0.33 units; output rises by less than G because both C and I fall.

The fall in investment after the government spending shock is not consistent with the steady state analysis, which showed that in the long run investment must be higher if government spending is higher. It is useful to think about the mechanism by which government spending impacts output in this model. When G goes up, the household feels poorer (because it has to pay more in taxes, either now or in the future). This makes it want to consume less and work more. Working more raises output. So the mechanism through which government spending impacts output in this model is not by stimulating "demand," but rather through a wealth effect channel wherein people feel poorer and supply more labor.

Using our logic from phase diagrams, we know that if the change in government spending is very transitory, this is very little wealth effect, and consumption should not change by much. If consumption does not jump by much, then the labor supply curve does not change by much, so there is little change in equilibrium hours, N_t. But if hours don't change by much, output doesn't change by much. From the aggregate resource constraint, if Y_t and C_t don't change by much, mechanically I_t must fall by approximately the increase in G_t. Effectively, for a very transitory change, a shock to government spending will just crowd out private investment one-for-one. As the shock to government spending gets more persistent, the wealth effect gets bigger – consumption would jump down by more in a phase diagram, which means labor supply would shift out by more, and hence Y_t would rise by more. But Y_t rising by more, and C_t falling by more in response to the same change in government spending means that I_t will fall by less. If the change in G_t is permanent, we would expect I_t to actually rise – this is because we know that K must be higher in the new steady state, so investment will have to go up to get us to transition to that new higher steady state.

So we would expect the persistence of changes in government spending, governed by the magnitude of ρ_g, to have very important effects on the equilibrium response to change in government spending. In particular, the bigger ρ_g is, we'd expect (i) output to rise by more, (ii) consumption to fall by more, (iii) hours to rise by more, and (iv) investment to fall by less, and for some sufficiently high value of ρ_g to actually rise. Below are impulse responses to a change in government spending for three different values of ρ_g:



The responses are exactly in line with what we would expect – Y rises by more, N by more, and C falls by more the more persistent is the change in government spending. I falls by less the more persistent is the change in government spending, and as we see for $\rho_g = 0.99$ investment actually rises.

3 Distortionary Taxes

Instead of assuming that taxes are all lump sum, I now all for distortionary tax rates on capital and labor income, τ_t^k and τ_t^n. I'll assume that there is an exogenous component to the tax rates, so that we can analyze the equilibrium effects of changes in tax rates. I'll consider two setups: one in which the government can still use lump sum taxes, and other in which there are no lump sum taxes. In the setup with lump sum taxes, the tax rates will show up in the equilibrium conditions, but debt will not again. When I prohibit the use of lump sum taxes we get more interesting stuff.

3.1 Tax Shocks, Lump Sum Finance

We assume that the government now can finance its spending decisions via issuing debt, lump sum taxes, and distortionary tax rates on labor and capital income on households. The household's budget constraint is now:

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} - B_t \leq (1-\tau_t^n)w_t N_t + (1-\tau_t^k)R_t K_t + \Pi_t - T_t + r_{t-1}B_t$$

τ_t^n and τ_t^k are potentially time-varying tax rates on labor and capital income, respectively. The first order conditions for the household problem can now be written:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \Big((1-\tau_{t+1}^k) R_{t+1} + (1-\delta) \Big) \right] \tag{23}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1 + r_t) \right] \tag{24}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} (1 - \tau_t^n) w_t \tag{25}$$

The firm problem is unaffected, so the FOC are to higher capital and labor up until the point at which factor prices equal the marginal product.

The government's budget constraint is given by:

$$G_t + r_{t-1} D_t \leq \tau_t^k R_t K_t + \tau_t^n w_t N_t + T_t + D_{t+1} - D_t \tag{26}$$

In equilibrium, we require that $D_t = B_t$ and $D_{t+1} = B_{t+1}$. Since $\Pi_t = Y_t - w_t N_t - R_t K_t$, the aggregate resource constraint still boils down to:

$$Y_t = C_t + I_t + G_t \tag{27}$$

The equilibrium conditions are then:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left((1 - \tau_{t+1}^k) R_{t+1} + (1 - \delta) \right) \right] \tag{28}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1 + r_t) \right] \tag{29}$$

$$w_t = (1 - \alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{30}$$

$$R_t = \alpha A_t K_t^{\alpha - 1} N_t^{1 - \alpha} \tag{31}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} (1 - \tau_t^n) w_t \tag{32}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1 - \alpha} \tag{33}$$

$$Y_t = C_t + I_t + G_t \tag{34}$$

$$K_{t+1} = I_t + (1 - \delta) K_t \tag{35}$$

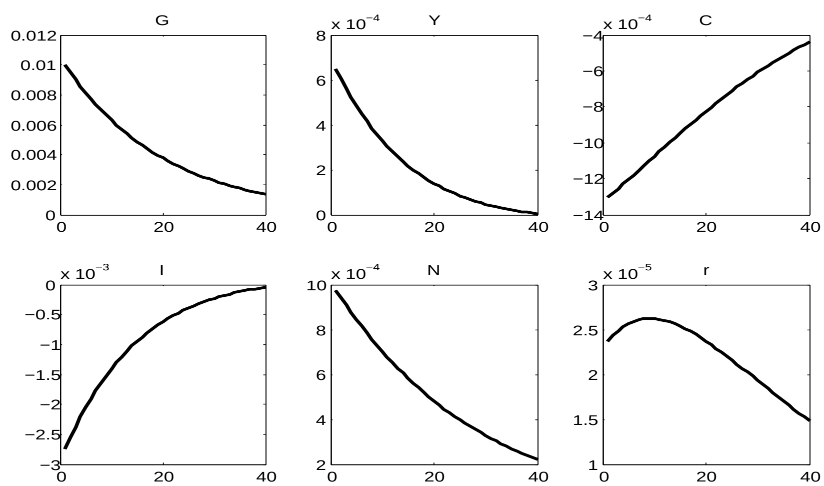
These are the same as we had before, with the exception that the capital tax rate shows up in the Euler equation for capital and the labor tax rate shows up in the first order condition for labor supply. We assume that both G_t and A_t follow the same log AR(1) processes given above. I assume that the tax rates obey stationary AR(1) processes with shocks, with tax rates without a time subscript denoting exogenous steady state values:

$$\tau_t^k = (1 - \rho_k) \tau^k + \rho_k \tau_{t-1}^k + \epsilon_{k,t} \tag{36}$$

$$\tau_t^n = (1 - \rho_n) \tau^n + \rho_n \tau_{t-1}^n + \epsilon_{n,t} \tag{37}$$

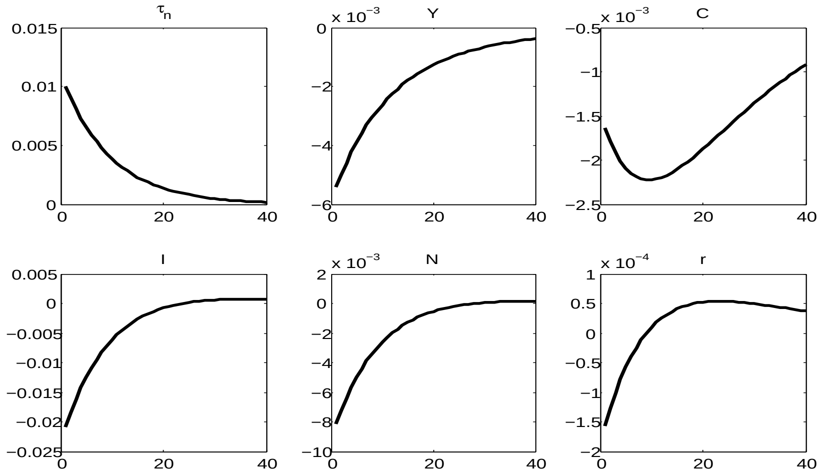
Even though we have added distortinary taxes, we again here have the result that D_t and T_t do not appear in the equilibrium conditions. It is therefore without loss of generality to assume that the government issues no debt and that T_t simply adjusts so as to make the government budget constraint hold every period. The model is nevertheless different in important ways with the presence of distortionary taxes. For example, since w_t, N_t, R_t, and K_t show up in the government's budget constraint, there is an endogenous reaction of government revenue to a change in government spending – it's not just all coming through an indeterminate mix between lump sum taxes and debt. This means that the impulse responses to a government spending shock will not necessarily be the same here as in the model with lump sum taxes only.

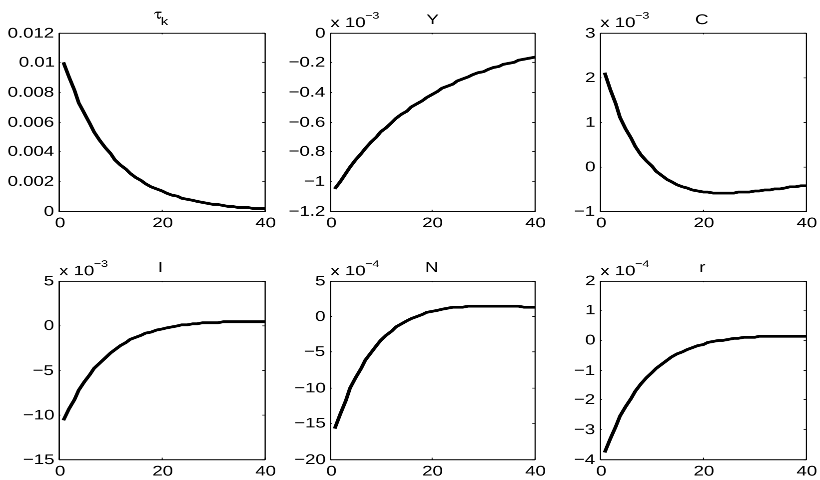
I compute impulse responses to shocks using the parameterization from the previous section (with $\rho_g = 0.95$), and assume that $\rho_n = \rho_k = 0.90$. I also assume that the standard deviations of the two tax shocks are 0.01. I assume that the steady state labor income tax is 0.2 and the steady state capital tax is 0.1. Below are impulse responses to a government spending shock.



Qualitatively, these look similar to what we had before. Y_t rises by roughly the same amount as earlier – the government spending multiplier now 0.3256, or just a little bit higher than what we had before.

Below are impulse responses to the two tax shocks:





We see that output, hours, consumption, and investment all decline when the tax rate on labor income goes up. People want to work less because they get to keep a smaller fraction of their labor; they want to consume less because the higher tax rate is like a negative shock to their wealth. Investment goes down because the marginal product of capital is lower with less employment.

The responses to the capital tax shock are a little bit different. When the capital tax rate goes up output goes down, but consumption rises. The higher tax on capital income encourages people to save less (so consume more) via a substitution effect. Hence investment goes down. In a mechanical sense, consuming more pushes labor supply in, so N falls and therefore output falls too.

3.2 No Lump Sum Taxes

Now let's continue to assume that there are distortionary tax rates on labor and capital income, but let's not allow the government to use lump sum taxes. Without lump sum taxes, government debt is not going to drop out of the equilibrium conditions.

The household and firm first order conditions are the same as before. The government's budget constraint without lump sum taxes is now:

$$G_t + r_{t-1}D_t \leq \tau_t^k R_t K_t + \tau_t^n w_t N_t + D_{t+1} - D_t \tag{38}$$

We will still require that $D_t = B_t$ and $D_{t+1} = B_{t+1}$ in equilibrium; hence the aggregate resource constraint will still be $Y_t = C_t + I_t + G_t$. But we will not be able to ignore the government's budget constraint in the equilibrium conditions. The reason we could do so earlier is that either T_t or D_{t+1} would adjust to make it hold; it didn't matter which, and the levels of T_t or D_t didn't affect anything else. This will no longer be the case. To see that, it is easiest to think about the government's budget constraint in the steady state.

Let's suppose that the government sets an exogenous long run (e.g. steady state) target for the debt-GDP ratio. This, combined with the long run level of government spending, will have bearing on the level of tax rates in the long run. Let's evaluate the government budget constraint at equality in the "long run" (e.g. at steady state, which I denote with the absence of a time subscript). Dividing both sides by Y, we have:

$$\frac{G}{Y} = \tau^k \frac{RK}{Y} + \tau^n \frac{wN}{Y} - r\frac{D}{Y}$$

Let's assume that $\frac{G}{Y} = \omega$ and that $\frac{D}{Y}$ is also exogenously given. We know that $r = \frac{1}{\beta} - 1$ from the household's Euler equation for bonds. Given assumptions on the production function plus the fact that factors are paid their marginal products, we know that $\frac{RK}{Y} = \alpha$ and $\frac{wN}{Y} = 1 - \alpha$. Using this we have:

$$\omega + \left(\frac{1}{\beta} - 1\right)\frac{D}{Y} = \alpha\tau^k + (1-\alpha)\tau^n \tag{39}$$

This must hold in the long run; if we make ω and $\frac{D}{Y}$ exogenous parameters, this means that we cannot freely choose the steady state values of both τ^k and τ^n. They must be set such that this constraint holds.

We can also see from this equation why a large debt-GDP ratio is costly. Since $\frac{1}{\beta} - 1$ is positive, a bigger debt-gdp ratio necessitates some combination of higher tax rates – either τ^k or τ^n have to be bigger, or both. These higher tax rates lower the steady state level of output and also lower welfare. Alternatively, if we wanted to fix tax rates, higher debt-gdp would have to be met by a lower share of government spending in output; this may also be undesirable if households get utility from government spending. The bottom line here is that the higher is the steady state debt-gdp ratio, you must have some combination of higher taxes or lower spending for the government's budget constraint to hold.

We also need to worry about debt and its relation with taxes outside of the steady state. There will in general not exist an equilibrium with a non-explosive path of government debt if we simply assume that government spending and the two tax rates follow exogenous processes. To ensure stability of debt, we need to write down processes which have some degree of tax rates rising in response to debt being too high relative to steady state (or government spending falling in response to debt being higher than steady state). I'm going to assume that this comes in through the tax rates, with government spending following the same exogenous AR(1) process in the log as before. I write down processes for the tax rates as follows:

$$\tau_t^k = (1-\rho_k)\tau^k + \rho_k \tau_{t-1}^k + (1-\rho_k)\gamma_k \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right) + \epsilon_{k,t} \tag{40}$$

$$\tau_t^n = (1-\rho_n)\tau^n + \rho_n \tau_{t-1}^n + (1-\rho_n)\gamma_n \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right) + \epsilon_{n,t} \tag{41}$$

These processes are the same as we had before, except for the coefficients γ_k and γ_n multiplying the deviation of the debt-gdp ratio from steady state. We require that these coefficients be such that debt be non-explosive in equilibrium – this will require that one or both of these coefficients be sufficiently positive so that debt doesn't explode upward (but also not so positive that debt doesn't explode downward). I pre-multiply both these coefficients by one minus the respective AR parameter because this gives these coefficients a "long run" interpretation – if the debt-GDP ratio were to permanently rise by 0.01, then the tax rate would permanently rise by γ_k or γ_n times this rise.

The full set of equilibrium conditions, excluding the exogenous process for A_t and G_t, are:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}}\Big((1-\tau_{t+1}^k)R_{t+1} + (1-\delta)\Big)\right] \tag{42}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}}(1+r_t)\right] \tag{43}$$

$$w_t = (1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha} \tag{44}$$

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{45}$$

$$\theta N_t^{\chi} = \frac{1}{C_t}(1-\tau_t^n)w_t \tag{46}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{47}$$

$$Y_t = C_t + I_t + G_t \tag{48}$$

$$K_{t+1} = I_t + (1-\delta)K_t \tag{49}$$

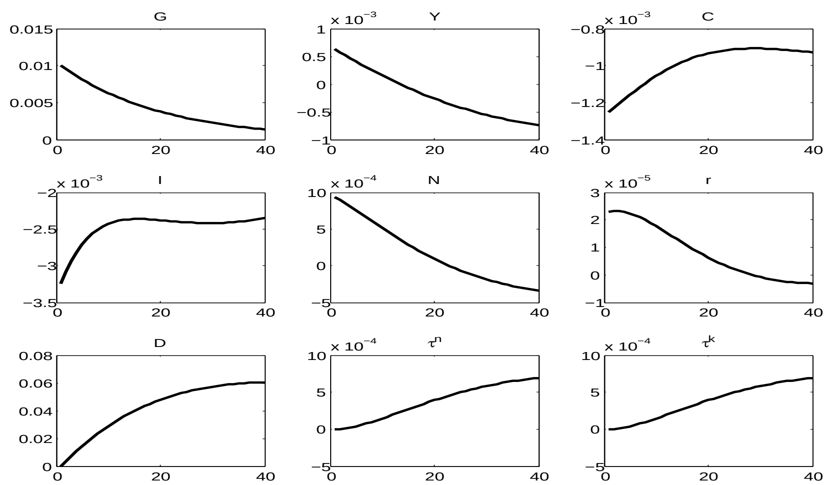
$$G_t + r_{t-1}D_t \leq \tau_t^k R_t K_t + \tau_t^n w_t N_t + D_{t+1} - D_t \tag{50}$$

$$\tau_t^k = (1-\rho_k)\tau^k + \rho_k \tau_{t-1}^k + (1-\rho_k)\gamma_k \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right) + \epsilon_{k,t} \tag{51}$$

$$\tau_t^n = (1-\rho_n)\tau^n + \rho_n \tau_{t-1}^n + (1-\rho_n)\gamma_n \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right) + \epsilon_{n,t} \tag{52}$$

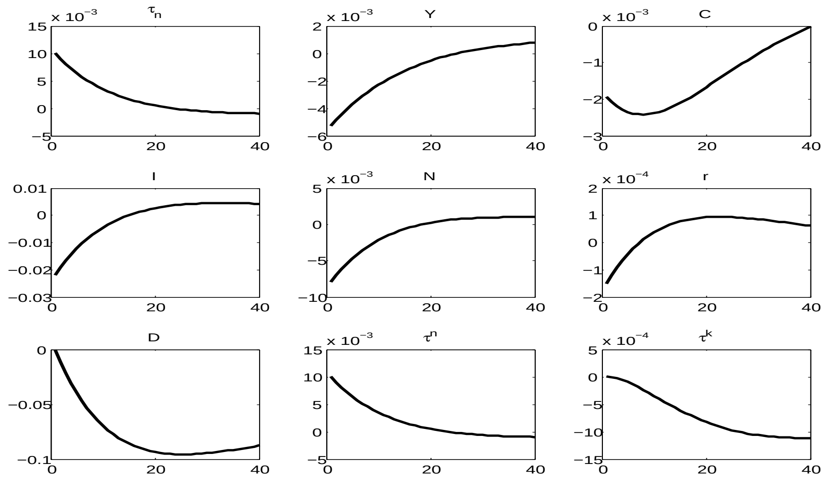
These are the same as we had before, except now I have the deviation of the debt-gdp ratio from steady state in the two tax processes. Given that I've added another endogenous variable, D_t, I have to include the government budget constraint as an equilibrium condition.

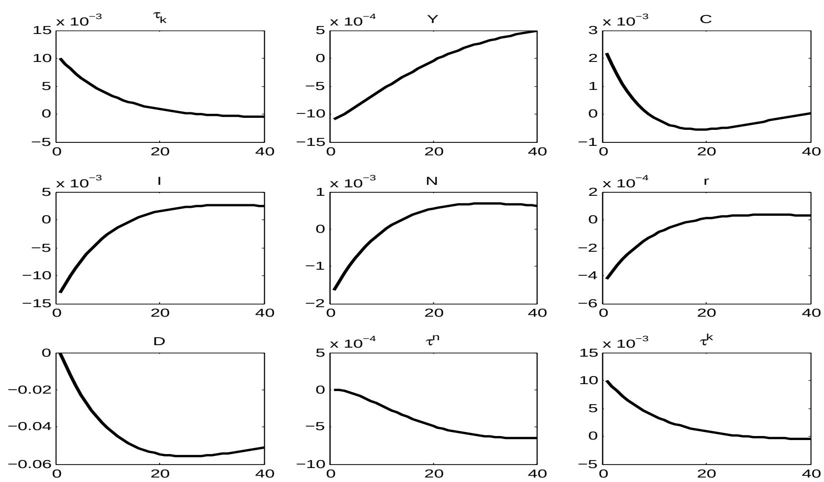
I use the same parameter value as before, this time setting $\gamma_k = \gamma_n = 0.025$. I assume that the steady state debt-gdp ratio is $\frac{D}{Y} = 0.5$. The impulse responses are shown below:



First, let's look at the impulse responses to a government spending shock. The immediate impact effects on endogenous variables are fairly similar to what we had before – here the output multiplier is 0.3131, so very similar to what we had earlier. But the responses out at longer forecast horizons are quite different. As we see in the lower left hand corner, the increase in government spending causes debt to rise. After a few periods, this triggers increases in both capital and labor taxes, which exert contractionary effects on overall economic activity. After several periods, the increase in government spending has mostly faded away, but tax rates are high, so output is actually lower than where it started before the shock.

Next, consider the impulse responses to the tax shocks.





In both cases, the immediate impact effects are pretty similar to what we had before, but again the responses are different in important ways out at further forecast horizons. The increase in either tax rate causes debt to fall. Falling debt automatically works to lower tax rates out at some horizon, which is relatively stimulative, so we see output eventually higher than where it started in both cases after a sufficiently long forecast horizon.

There are a lot of different experiments you could run here – you can fiddle with the γ and ρ coefficients to generate different debt-dynamics, which can induce pretty interesting dynamics for the rest of the variables in the model.

4 Optimal Taxation in the Long Run

Rather than focusing on short run equilibrium dynamics assuming exogenous values of taxes and spending, let's briefly think about optimal taxes and government spending in the long run (e.g. the steady state).

Consider the model where we do not permit the government to use lump sum taxes (otherwise the problem of choosing optimal tax rates is pretty easy – it doesn't want to have those taxes). We will focus on the steady state. Suppose that steady state flow utility for the representative household is given by:

$$U = \ln C - \theta \frac{N^{1+\chi}}{1+\chi} + \xi \ln G$$

While it is not necessary to think about utility from government spending in terms of equilibrium dynamics (so long as utility from government spending is additively separable), for thinking about welfare it is important.

Taking the steady state government spending-output ratio, ω, and the steady state debt-gdp ratio, $\frac{D}{Y}$, as given, we can solve for the steady state values of C and N in terms of tax rates as:

$$\frac{K}{N} = \left(\frac{(1-\tau^k)\alpha}{\frac{1}{\beta} - (1-\delta)} \right)^{\frac{1}{1-\alpha}} \tag{53}$$

$$\frac{C}{N} = (1-\omega)\left(\frac{K}{N}\right)^{\alpha} - \delta \frac{K}{N} \tag{54}$$

$$N = \left(\frac{1}{\theta} \frac{(1-\tau^n)(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}}{(1-\omega)\left(\frac{K}{N}\right)^{\alpha} - \delta\frac{K}{N}} \right)^{\frac{1}{1+\chi}} \tag{55}$$

$$Y = \left(\frac{K}{N}\right)^{\alpha} N \tag{56}$$

$$C = \frac{C}{N} N \tag{57}$$

$$G = \omega Y \tag{58}$$

In steady state, the tax rates must satisfy:

$$\omega + \left(\frac{1}{\beta} - 1\right)\frac{D}{Y} = \alpha\tau^k + (1-\alpha)\tau^n \tag{59}$$

I'm going to do a couple of experiments. In particular, I first want to find the steady state tax rates on capital and labor, τ^k and τ^n, which maximize steady state utility, taking $\frac{D}{Y}$ and ω as given. In doing this, I want to impose a non-negativity constraint – the tax rates cannot be negative. In principle, I could do this analytically, but it's pretty laborious. So I do it numerically by searching over τ^k and τ^n to maximize U subject to all of the above conditions holding, as well as the restriction that the tax rates be non-negative. I assume that $\frac{D}{Y} = 0.5$ and $\omega = 0.2$, and use the other parameter values with which I've been working. I set $\xi = 0.3$.

I find that the optimal steady state tax rates are $\tau^n = 0.3076$ and $\tau^k = 0.0000$; this results in a steady state utility level of -1.0349 (as opposed to -1.0923 when I fixed $\tau^n = 0.20$ and find the steady state capital tax rate consistent with the government budget constraint holding). Note that the utility level being negative is fine, as utility is ordinal; as we would expect, steady state utility is higher when I optmally choose the tax rates.

I do a second experiment in which I simultaneously choose τ^n and τ^k, but also choose the steady state government spending-gdp ratio, ω, instead of fixing it at 0.2. Here I find that the optimal government spending share is $\omega = 0.1679$ (a little lower than I assumed before), and the optimal tax rates are $\tau^n = 0.2621$ and $\tau^k = 0.0000$. The steady state utility level associated with these values is -1.0291. This is naturally higher than what I get when I fixed ω.

What we see coming out here is a result in the literature dating back to Chamley (1985) and Judd (1986). This literature states that in the steady state of a neoclassical growth model capital should not be taxed. Different intuitions have been offered for this result. My own is simple Econ 101 type reasoning. The standard optimal taxation literature says that you should tax different things depending inversely on how elastically they are supplied/demanded. If a good is in either inelastic supply or demand, then the equilibrium quantity of that good does not depend on the price of the good. Taxes just drive a wedge between the "buy" and "sell" prices of a good. But if equilibrium quantity doesn't depend on price, then a positive tax doesn't distort quantities produced – it represents a transfer from buyers to sellers (or vice versa depending on whether demand or supply are inelastic), but it doesn't actually distort a productive margin. For something which is in either completely inelastic supply or demand, a distortionary tax is isomorphic to a lump sum tax. The more elastic the supply or demand of a good is, the bigger are the quantity distortions from taxes. You can see this by drawing "deadweight" triangles in simple supply and demand graphs.

The simple insight with respect to capital is this. In the long run (e.g. the steady state), capital is in perfectly elastic supply. The long run rental rate on capital satisfies $(1 - \tau^k)R = \frac{1}{\beta} - (1 - \delta)$. This steady state rental rate depends on how the household discounts the future and how fast capital depreciates; it does not depend on the level of the capital stock itself. This relationship is the long run capital supply curve – it is perfectly elastic. Capital demand is downward-sloping in K and equates the marginal product of K with the rental rate. With perfectly elastic supply of capital, the deadweight losses from capital taxation are large. For labor, neither demand nor supply of labor are perfectly elastic. This means, intuitively, that the welfare losses from labor taxation are smaller than for capital taxation, which is why we see that it is apparently optimal to have a relatively low tax on capital income in the long run.

The interesting side note to this discussion is that this idea of zero capital tax only applies to the long run. Whereas capital is in perfectly elastic supply in the long run, in the short run capital is perfectly inelastic – it is pre-determined, after all. This means that if a government wanted to raise money in an efficient way in a dynamic sense, it should levy a very large tax on capital at the beginning of time with a promise to eliminate the capital tax in the infinite future. This is because capital is inelastic in the short run, so that a capital tax is functionally like a lump sum tax, whereas in the long run it is perfectly elastic and carries with it large distortions. Of course, there is time inconsistency issue at play here – it may be hard to commit to a high capital tax in the present and a low capital tax in the future.